

Introduction to Nonlinear Dynamical Systems and Nonlinear Control Strategies

Presenter: **Dr. Bingo Wing-Kuen Ling**

Lecturer, Centre for Digital Signal Processing,
Department of Electronic Engineering, King's
College London.

Email address: wing-kuen.ling@kcl.ac.uk

Outline

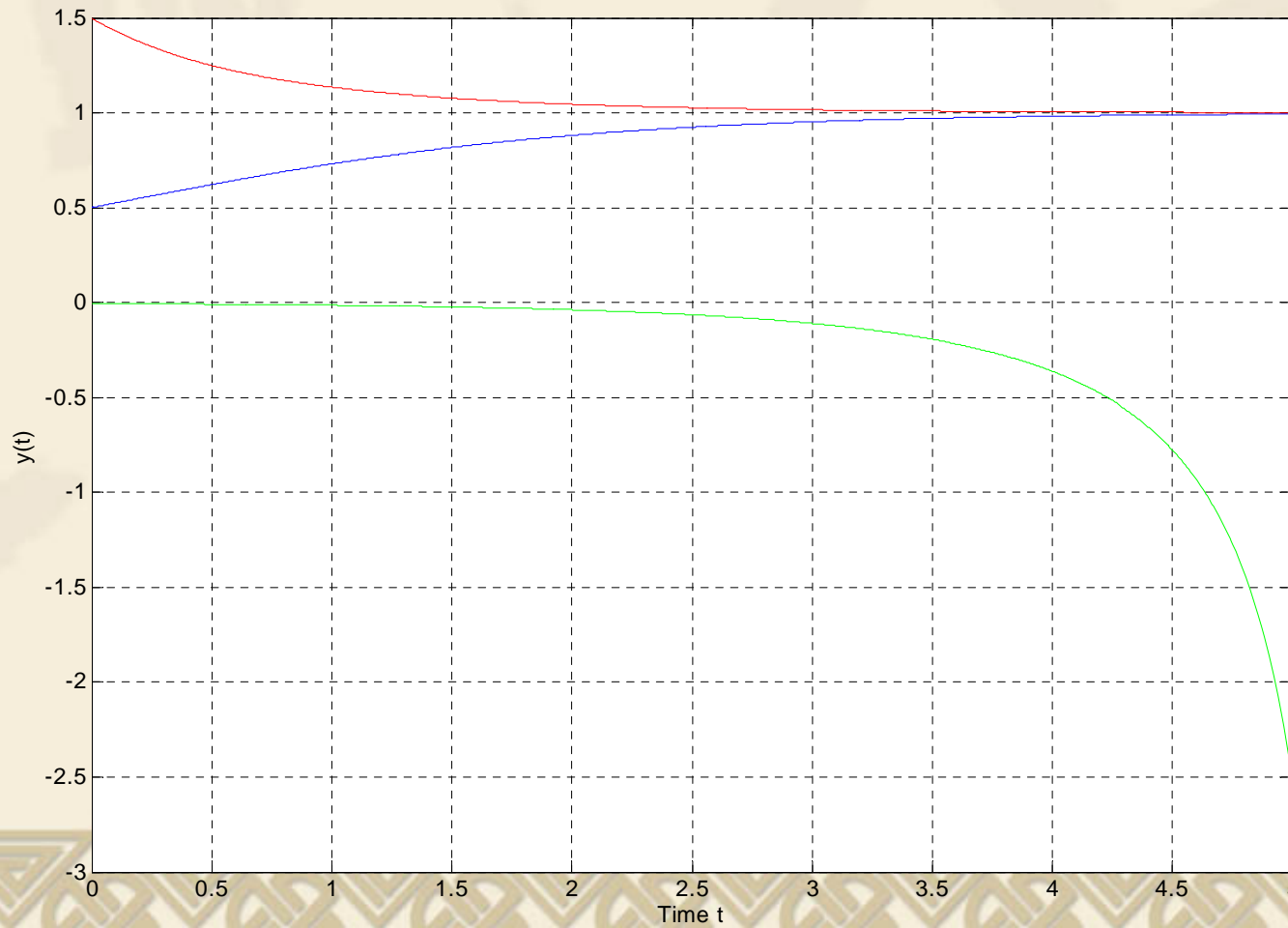
- ❖ Nonlinear Systems
 - ❧ Behaviors of nonlinear systems
 - ❧ Definition of chaos
 - ❧ Definitions of symbolic dynamics
 - ❧ Examples of systems governed by symbolic dynamics
- ❖ Challenge Problems and Some Solutions
 - ❧ Sigma delta modulators
 - ❧ Digital filters with two's complement arithmetic
 - ❧ Perceptrons
- ❖ Impulsive Control
- ❖ Fuzzy Control
- ❖ Conclusions
- ❖ Questions and Answers

Nonlinear Systems

❖ Behaviors of nonlinear systems

∞ Behaviors depend on the initial conditions.

$$\frac{dx(t)}{dt} = x(t)(1 - x(t))$$



Nonlinear Systems

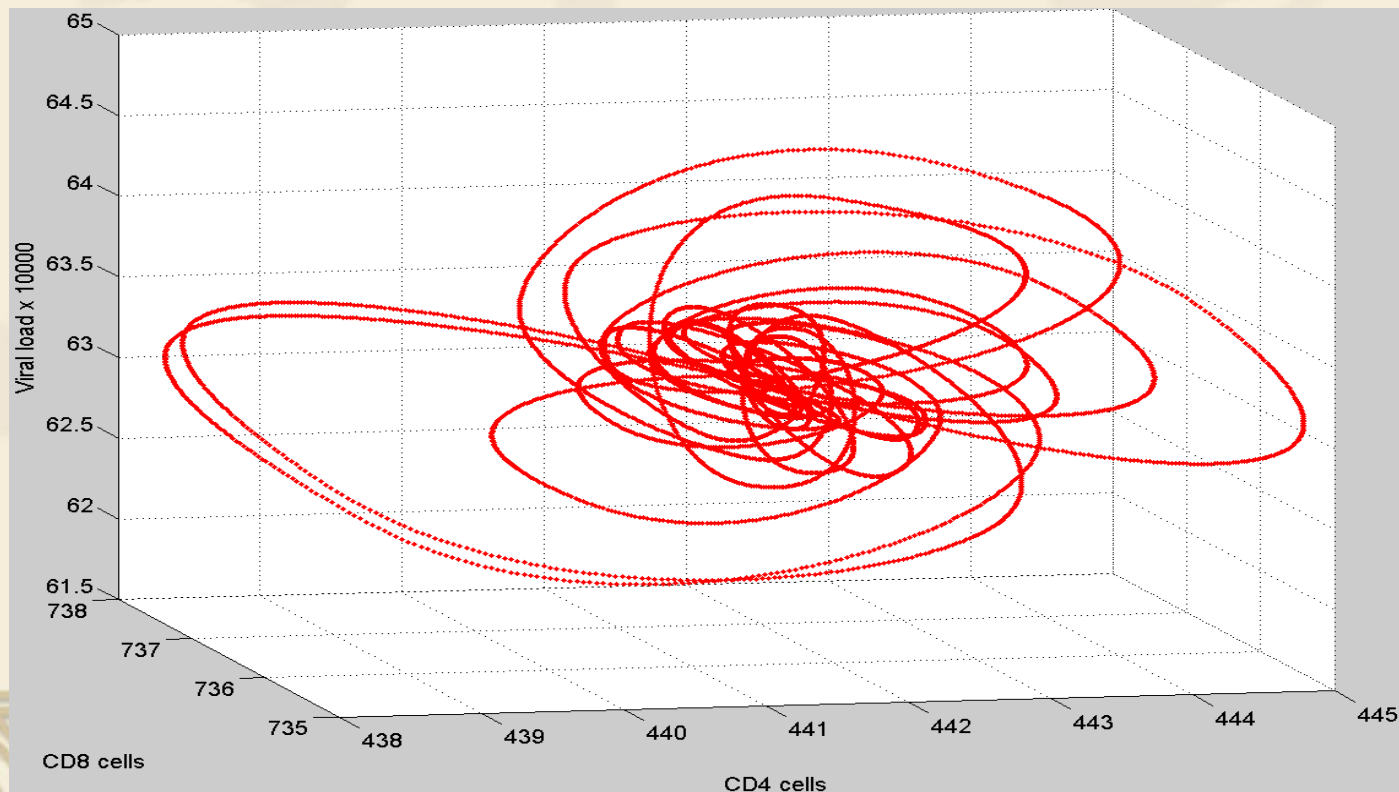
❖ Behaviors of nonlinear systems

∞ Behaviors could be very complicated.

$$\dot{x}(t) = a(x_0 - x(t)) - bx(t)z(t)$$

$$\dot{y}(t) = c(y_0 - y(t)) + dy(t)z(t)$$

$$\dot{z}(t) = z(t)(ex(t) - fy(t))$$



Nonlinear Systems

❖ Definition of chaos (Devaney)

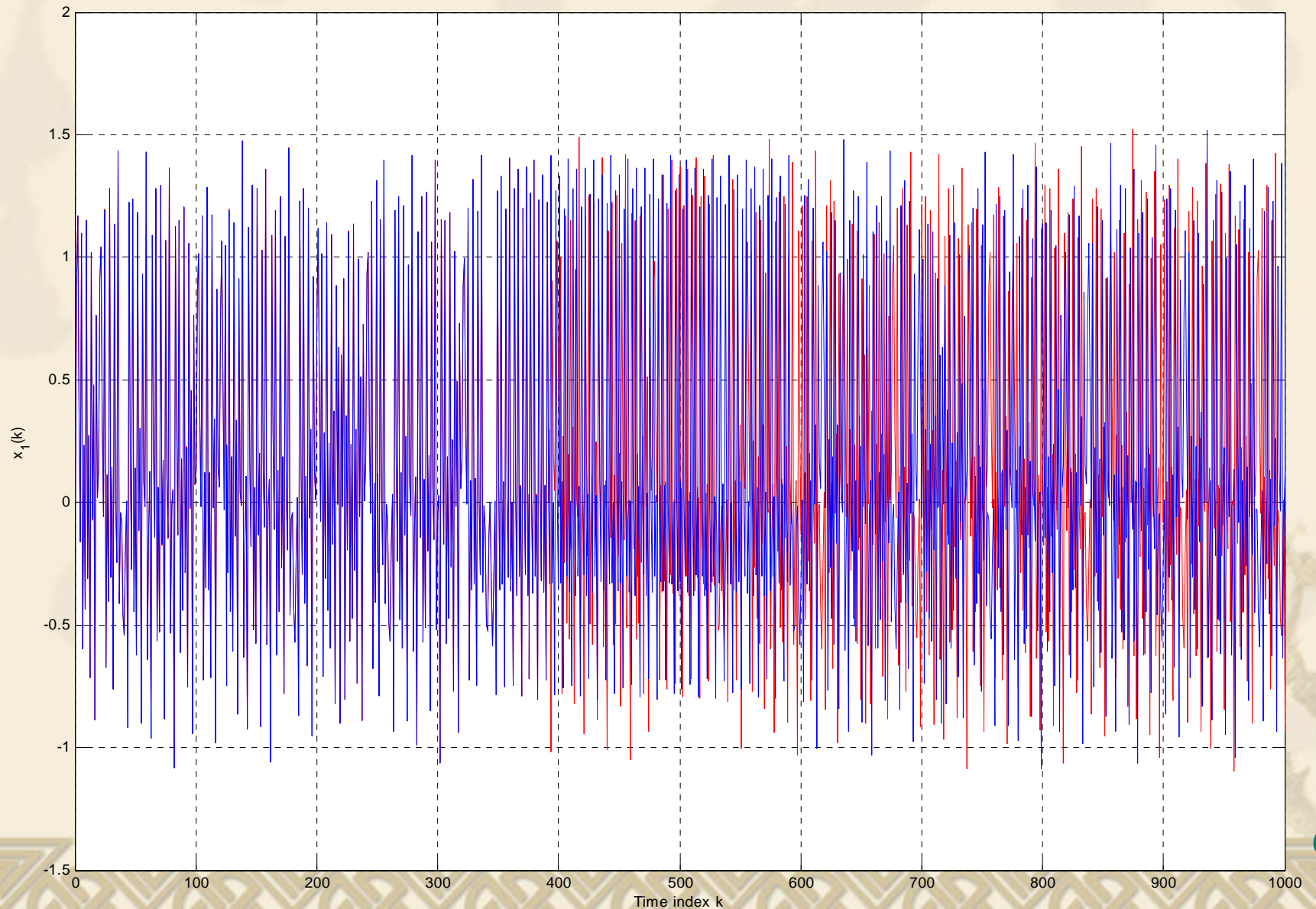
⌘ Sensitive to initial conditions

An interval map $F: I \rightarrow I$ has sensitive dependence to initial conditions if there exists a $\delta > 0$, such that, for any $x \in I$, and any neighborhood Ω of x , there exists $y \in \Omega$, and an iteration $n \geq 0$ such that

$$|F^n(x) - F^n(y)| > \delta.$$

⌘ That is, no matter how small the neighborhood, the map has the ability that a small error can grow terribly large. This is often referred to as the Butterfly effect, due to the thought-experiment described by Edward Lorenz.

Nonlinear Systems



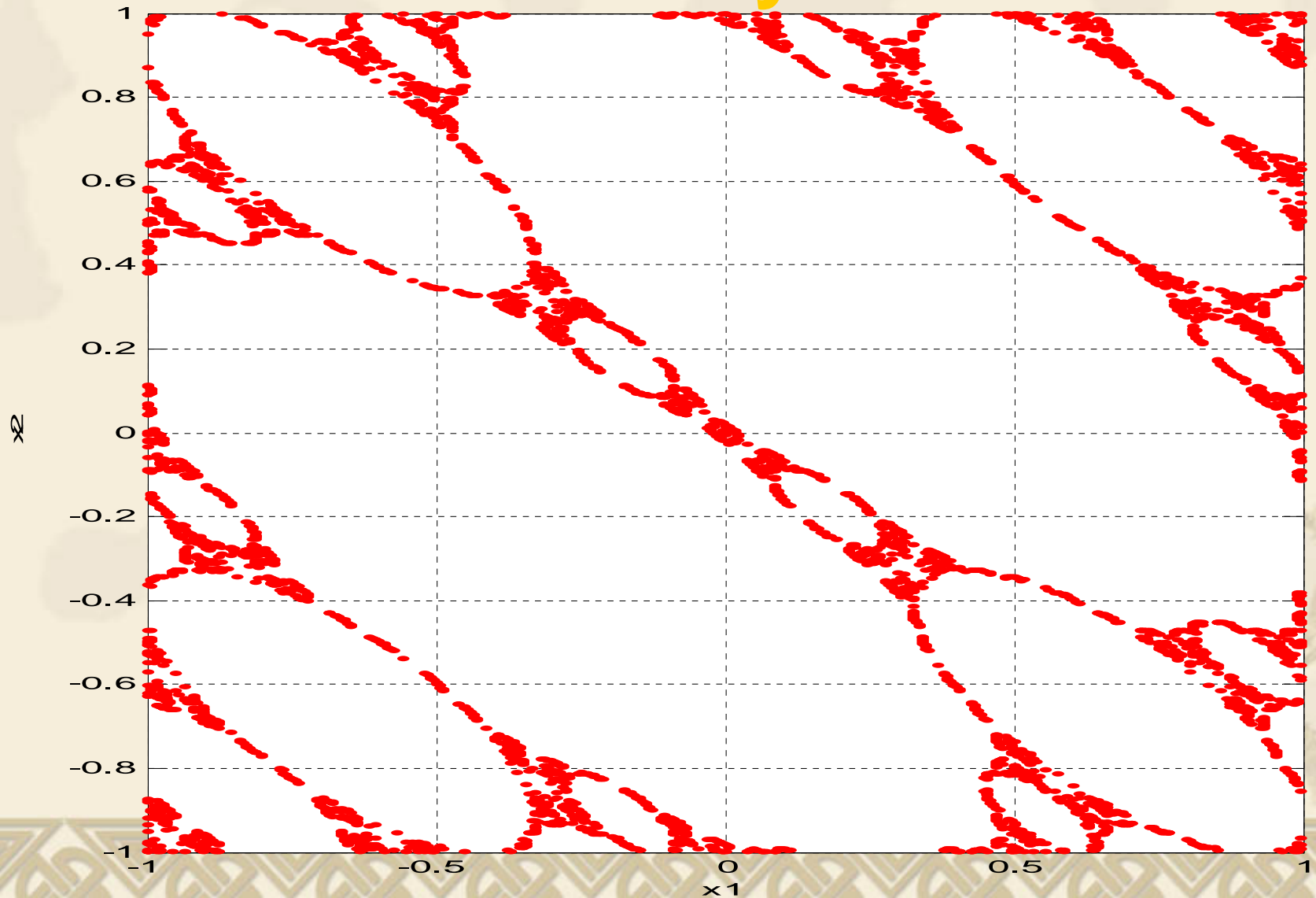
Nonlinear Systems

∞ Topological transitive

An interval map $F: I \rightarrow I$ is topologically transitive if for any two open sets $U, V \subset I$, there exists $k > 0$ such that $F^k(U) \cap V \neq \emptyset$.

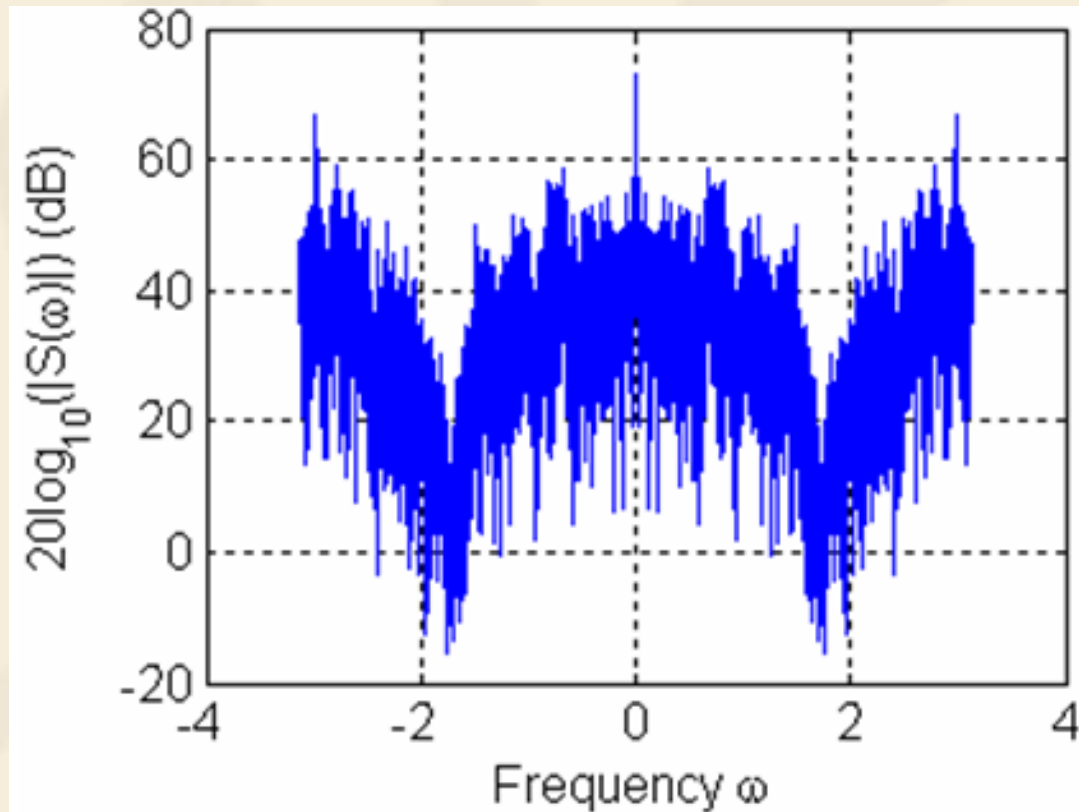
∞ This is equivalent to the statement that there exists a dense orbit. In other words, the state vector can end up almost anywhere in phase space.

Nonlinear Systems



Nonlinear Systems

∞ The periodic orbits are dense.



∞ Any 2 of the above forces the third to follow.

Nonlinear Systems

❖ Definition of symbolic dynamics

⌘ Symbolic dynamics is a kind of system dynamics that some signals in the system are multi-levelled.

⌘ Denote k as the time index, $\mathbf{x}(k)$ as the system state vector, $u(k)$ as the system input, $\mathbf{s}(k)$ as the symbolic vectors, S as a set of symbols. Then the system can be represented by the following state space equation:

$$\mathbf{x}(k+1) = f(\mathbf{x}(k), \mathbf{s}(k), \mathbf{u}(k), k)$$

where $\mathbf{s}(k) = g(\mathbf{x}(k)) \in S$

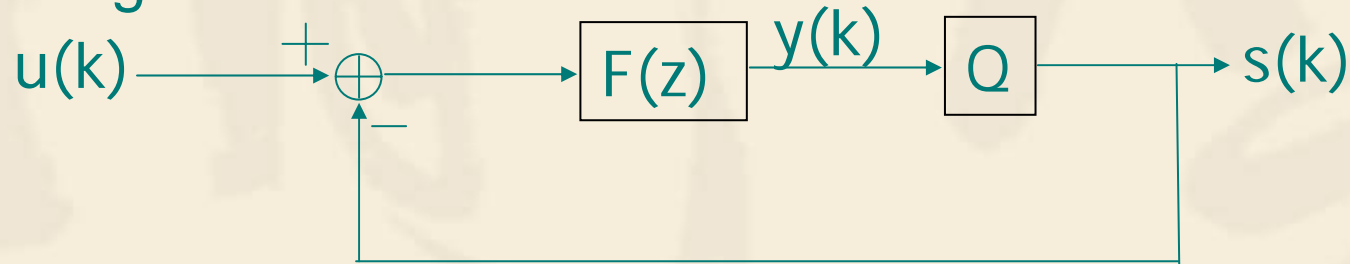
Nonlinear Systems

- ❖ Examples of systems governed by symbolic dynamics
 - ∞ Sigma delta modulators
 - ∞ Digital filters with two complement arithmetic
 - ∞ Perceptrons
 - ∞ Phase lock loops
 - ∞ Turbo decoders
 - ∞ etc...

Nonlinear Systems

❖ Examples of systems governed by symbolic dynamics

∞ Sigma delta modulators



Let

$$F(z) = \frac{\sum_{j=1}^N b_j z^{-j}}{\sum_{i=0}^N a_i z^{-i}}$$

Then

$$\sum_{i=0}^N a_i y(k-i) = \sum_{j=1}^N b_j (u(k-j) - s(k-j))$$

$$y(k) = \frac{1}{a_0} \sum_{j=1}^N b_j (u(k-j) - s(k-j)) - a_j y(k-j)$$

Nonlinear Systems

$$\begin{bmatrix} y(k-N+1) \\ \vdots \\ y(k-1) \\ y(k) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \dots & \dots & 0 & 1 \\ -\frac{a_N}{a_0} & \dots & \dots & \dots & -\frac{a_1}{a_0} \end{bmatrix} \begin{bmatrix} y(k-N) \\ \vdots \\ y(k-2) \\ y(k-1) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & \dots & \dots & \dots & 0 \\ \vdots & & & & \vdots \\ \vdots & & & & \vdots \\ 0 & \dots & \dots & \dots & 0 \\ \frac{b_N}{a_0} & \dots & \dots & \dots & \frac{b_1}{a_0} \end{bmatrix} \begin{bmatrix} u(k-N)-s(k-N) \\ \vdots \\ u(k-2)-s(k-2) \\ u(k-1)-s(k-1) \end{bmatrix}$$

Nonlinear Systems

Let $Q(y) \equiv \begin{cases} 1 & y \geq 0 \\ -1 & \text{otherwise} \end{cases}$

$$\mathbf{u}(k) \equiv [u(k-N), \dots, u(k-1)]^T$$

then $\mathbf{x}(k) \equiv [x_1(k), \dots, x_N(k)]^T \equiv [y(k-N), \dots, y(k-1)]^T$

$$\mathbf{s}(k) \equiv [s_1(k), \dots, s_N(k)]^T \equiv [Q(y(k-N)), \dots, Q(y(k-1))]^T$$

$$\mathbf{A} \equiv \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \dots & \dots & 0 & 1 \\ -\frac{a_N}{a_0} & \dots & \dots & \dots & -\frac{a_1}{a_0} \end{bmatrix}$$

Nonlinear Systems

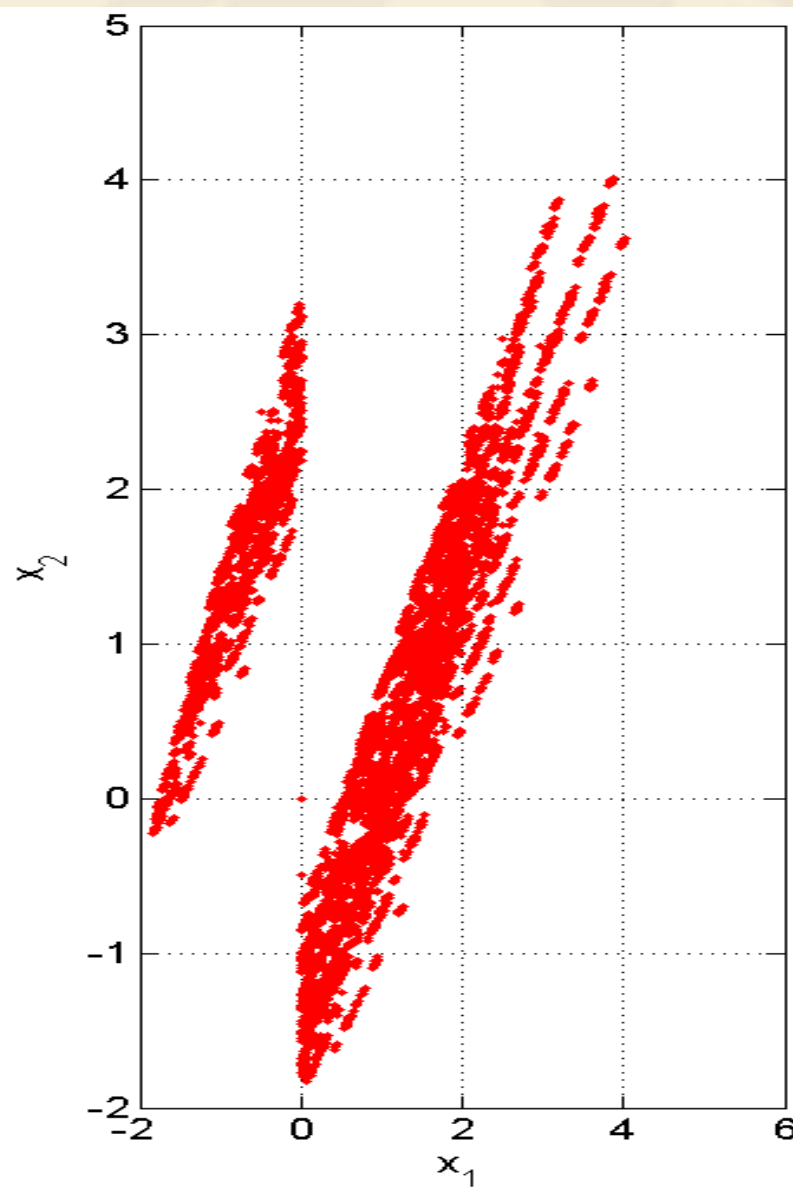
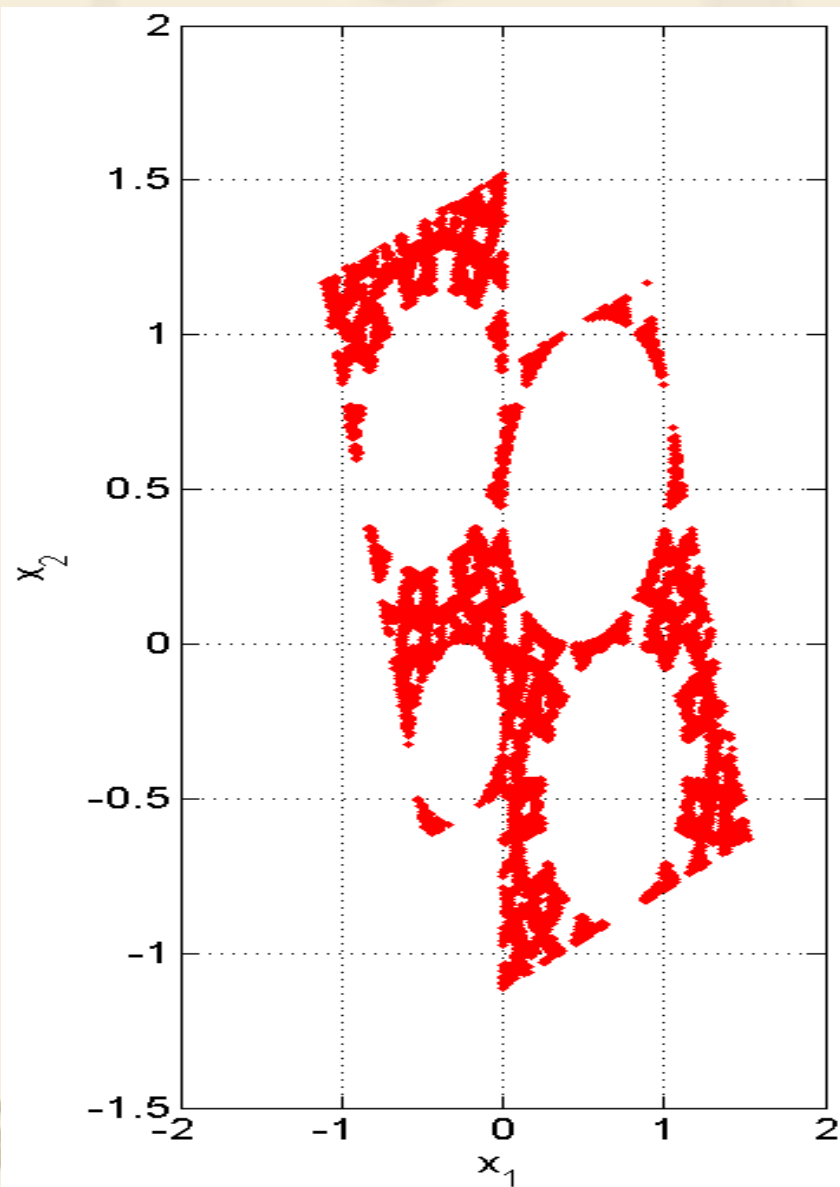
$$\mathbf{B} \equiv \begin{bmatrix} 0 & \dots & \dots & \dots & 0 \\ \vdots & & & & \vdots \\ \vdots & & & & \vdots \\ 0 & \dots & \dots & \dots & 0 \\ \frac{b_N}{a_0} & \dots & \dots & \dots & \frac{b_1}{a_0} \\ a_0 & & & & a_0 \end{bmatrix}$$

then $\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}(\mathbf{u}(k) - \mathbf{s}(k))$

where

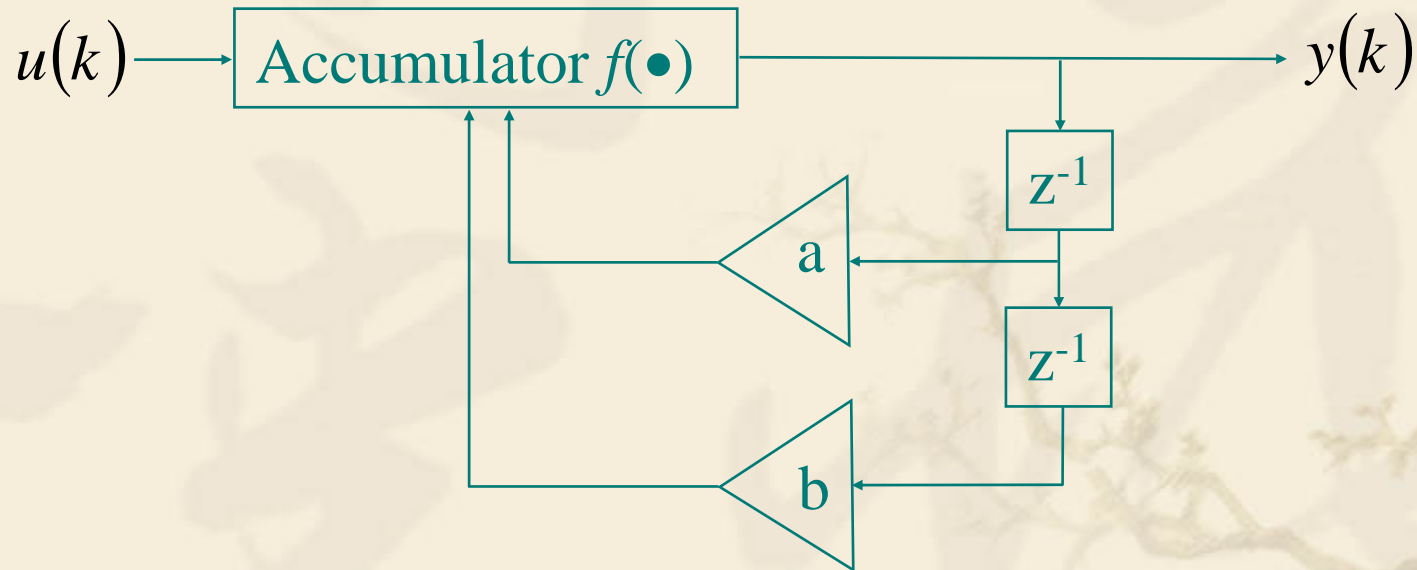
$$\mathbf{s}(k) = Q(\mathbf{x}(k)) \in \{-1, 1\} \times \{-1, 1\} \times \dots \times \{-1, 1\}$$

Nonlinear Systems



Nonlinear Systems

- ❖ Examples of systems governed by symbolic dynamics
 - ⌘ Digital filters with two complement arithmetic



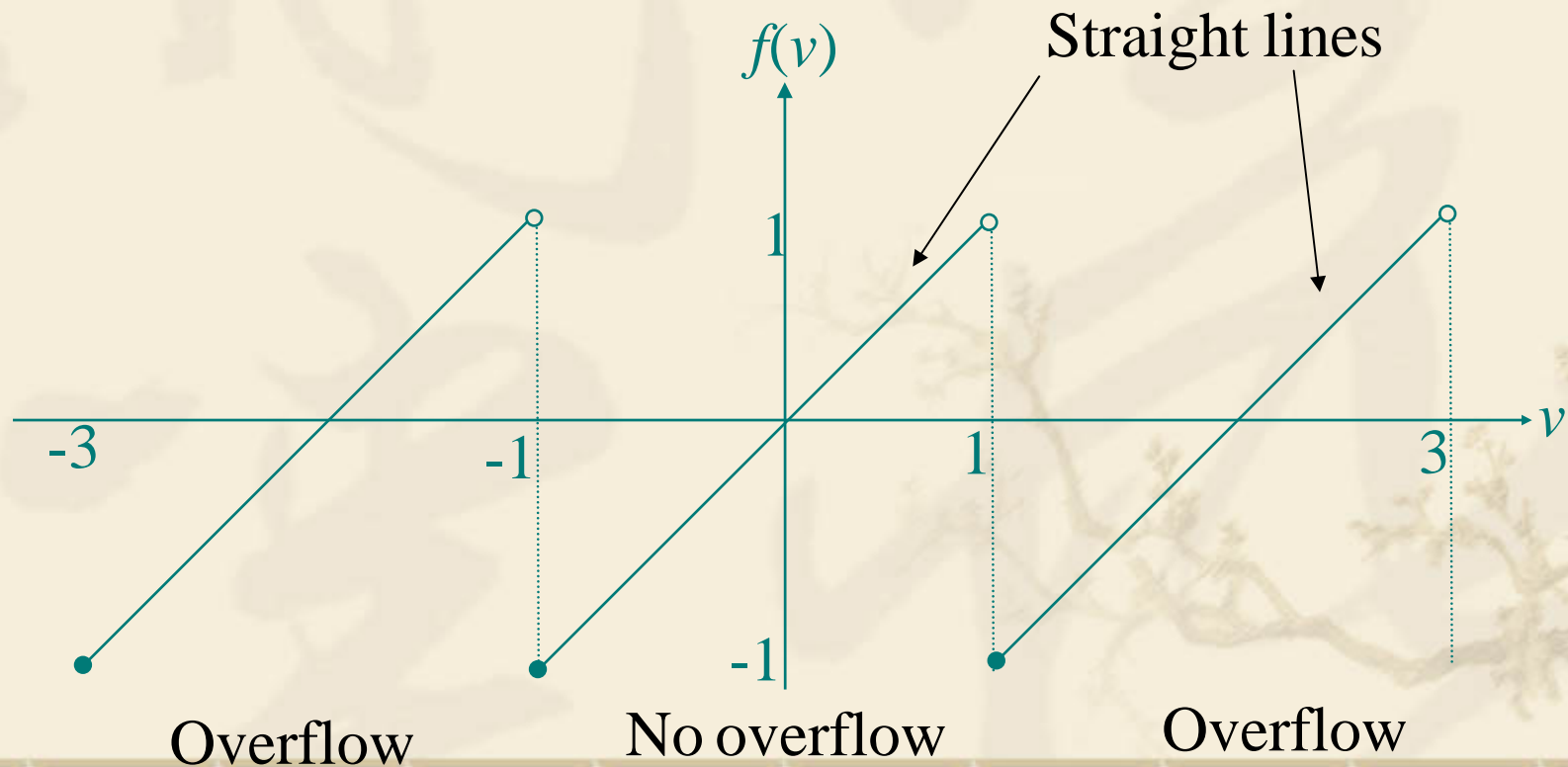
Nonlinear Systems

The digital filter can be described by the following nonlinear state-space difference equation:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} x_2(k) \\ f(bx_1(k) + ax_2(k) + u(k)) \end{bmatrix}$$

Nonlinear Systems

where $f(\bullet)$ is the nonlinear function associated with the two's complement arithmetic



Nonlinear Systems

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ b & a \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} s(k)$$

Let $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ b & a \end{bmatrix}$

$$\mathbf{x}(k) \equiv \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \in I^2 \equiv \{(x_1, x_2) : -1 \leq x_1 < 1, -1 \leq x_2 < 1\}$$

$$s(k) \in \{-m, \dots, -1, 0, 1, \dots, m\}$$

and m is the minimum integer satisfying

$$-2m-1 \leq bx_1(k) + ax_2(k) + u(k) \leq 2m+1$$

Then $\mathbf{x}(k+1) = \mathbf{A} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} s(k)$

Nonlinear Systems

$s(k)$ is called symbolic sequences.

$$s(k) = 1 \Rightarrow -3 \leq bx_1(k) + ax_2(k) + u(k) < -1$$

$$s(k) = 0 \Rightarrow -1 \leq bx_1(k) + ax_2(k) + u(k) < 1$$

$$s(k) = -1 \Rightarrow 1 \leq bx_1(k) + ax_2(k) + u(k) < 3$$

\vdots

Nonlinear Systems

For example:

$$\mathbf{x}(0) = [-0.6135 \quad 0.6135]^T, b = -1, a = 0.5 \text{ and } u(k) = 0$$

$$bx_1(0) + ax_2(0) = 0.9203 \Rightarrow s(0) = 0$$

$$bx_1(1) + ax_2(1) = -0.1534 \Rightarrow s(1) = 0$$

⋮

Two's complement value

$$bx_1(14) + ax_2(14) = 1.0018 \Rightarrow s(14) = -1 \text{ and } x_2(15) = -0.9982$$

$$bx_1(15) + ax_2(15) = -0.7597 \Rightarrow s(15) = 0$$

$$bx_1(16) + ax_2(16) = 0.6184 \Rightarrow s(16) = 0$$

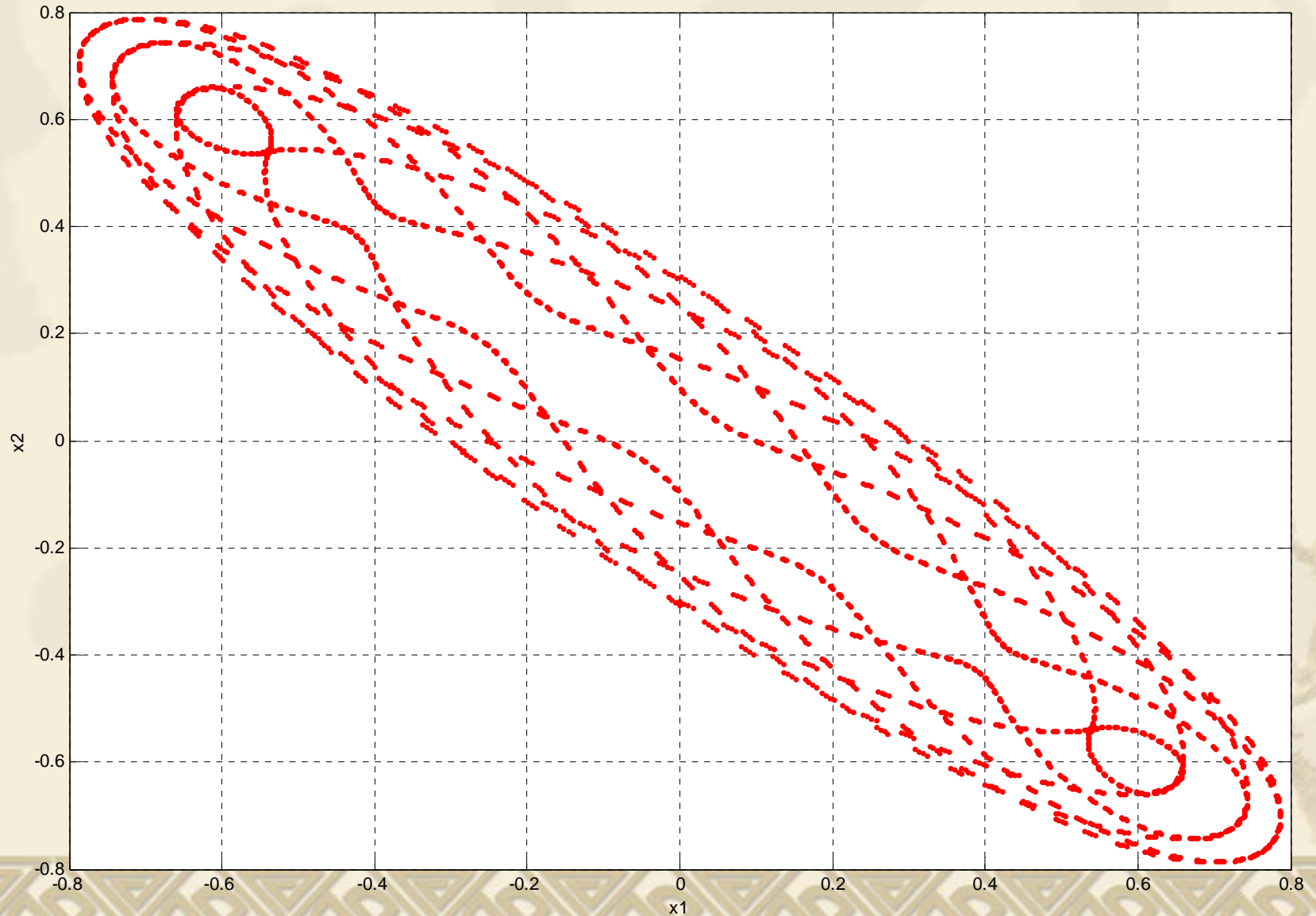
$$bx_1(17) + ax_2(17) = 1.0689 \Rightarrow s(17) = -1 \text{ and } x_2(18) = -0.9311$$

$$bx_1(18) + ax_2(18) = -1.0839 \Rightarrow s(18) = 1 \text{ and } x_2(19) = 0.9161$$

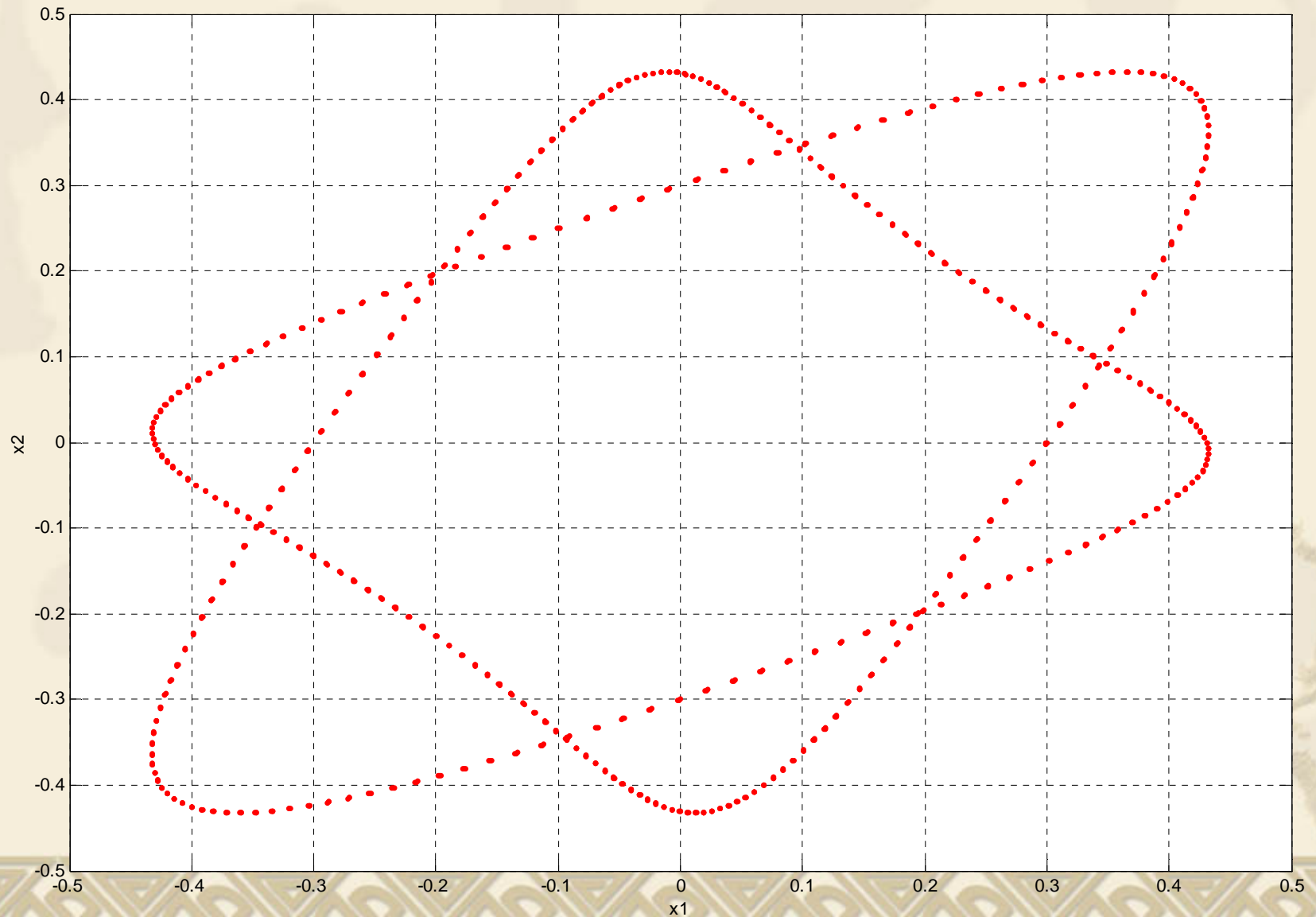
⋮

$$s = (0 \quad 0 \quad \dots \quad 0 \quad -1 \quad 0 \quad 0 \quad -1 \quad 1 \quad \dots)$$

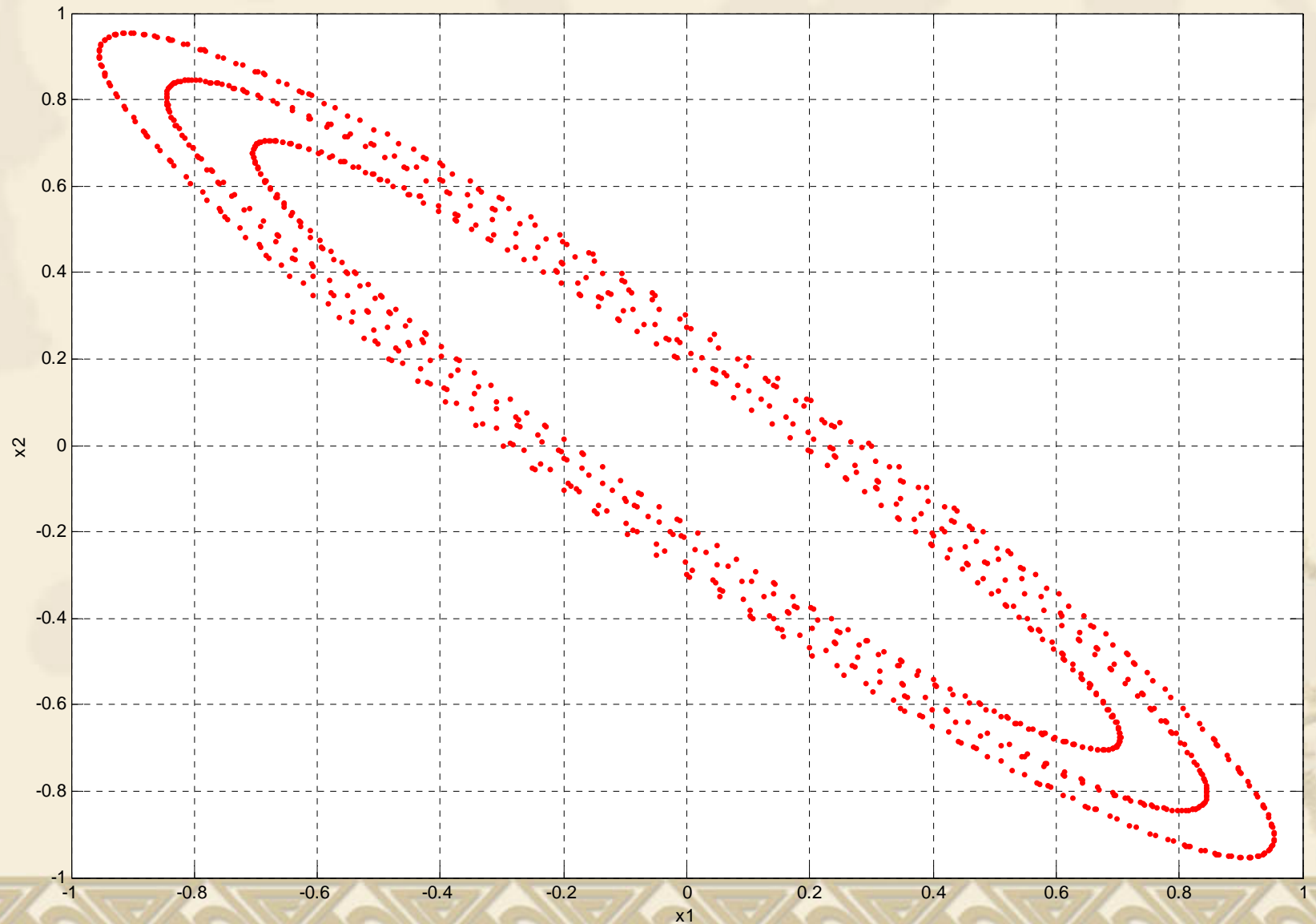
Nonlinear Systems



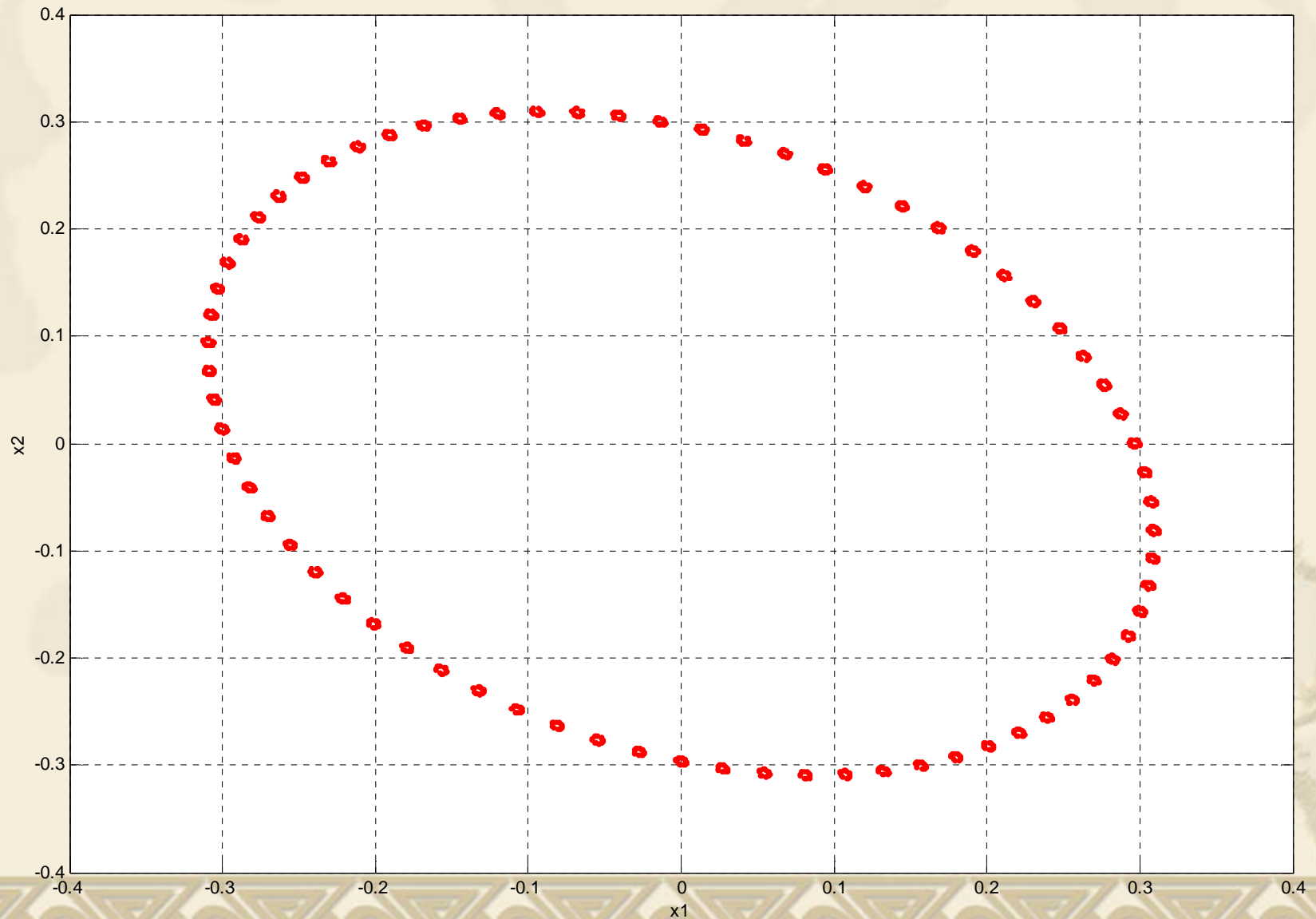
Nonlinear Systems



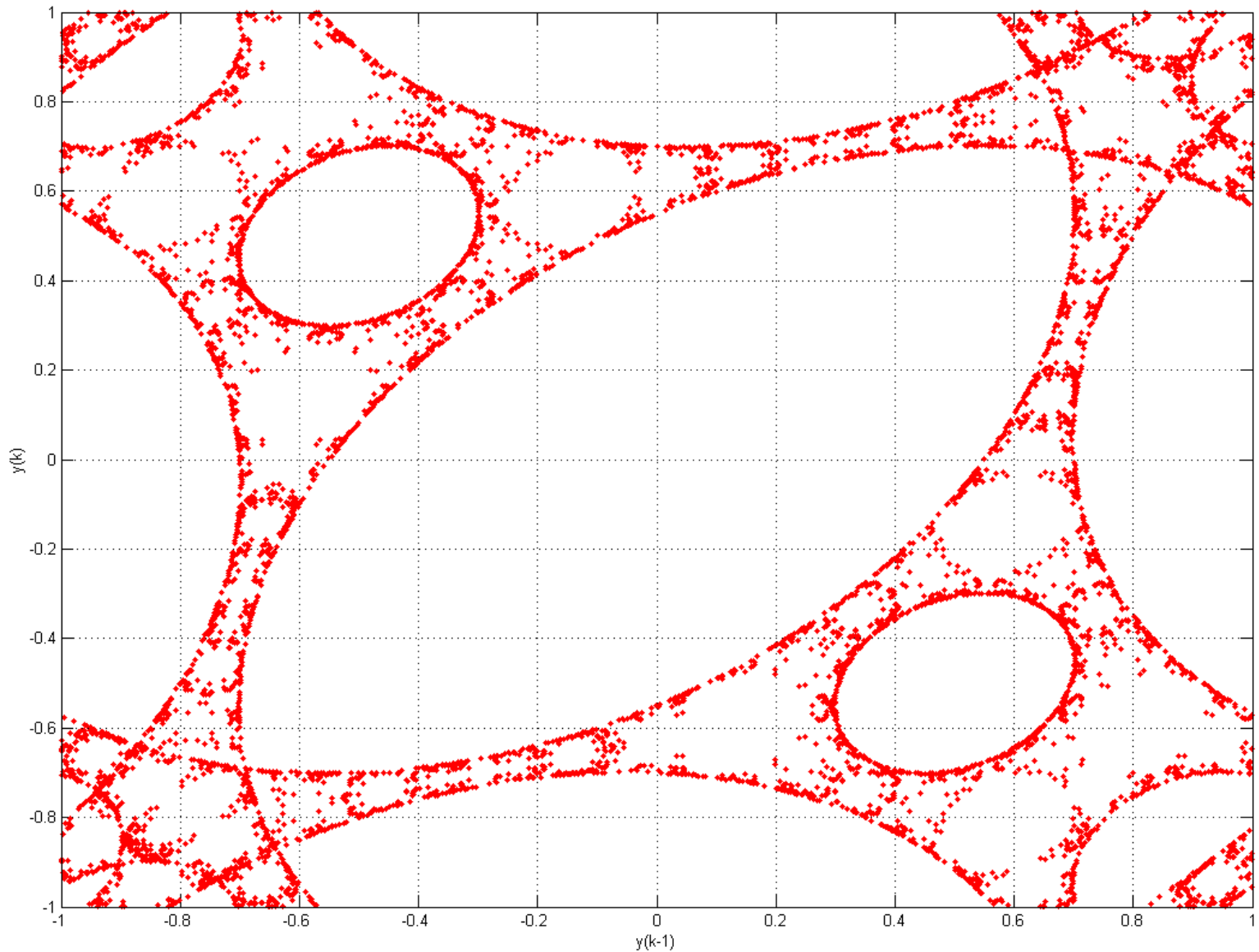
Nonlinear Systems



Nonlinear Systems



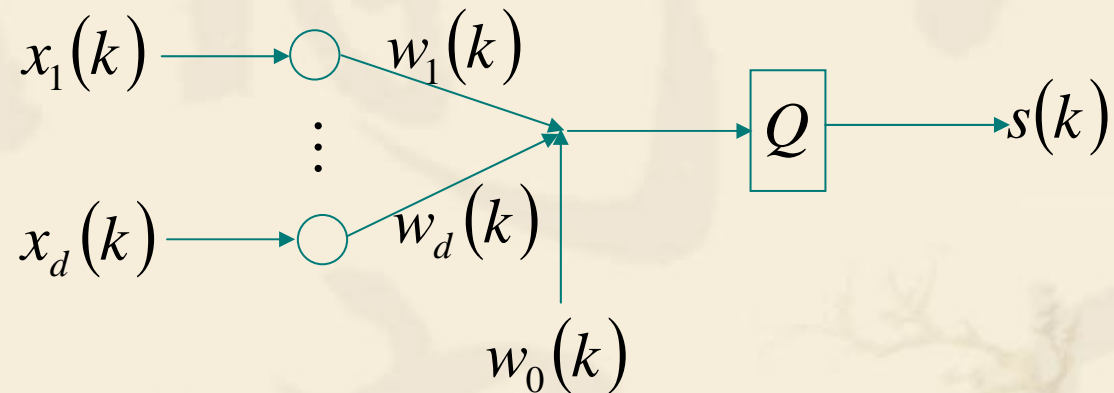
Nonlinear Systems



Nonlinear Systems

❖ Examples of systems governed by symbolic dynamics

∞ Perceptrons

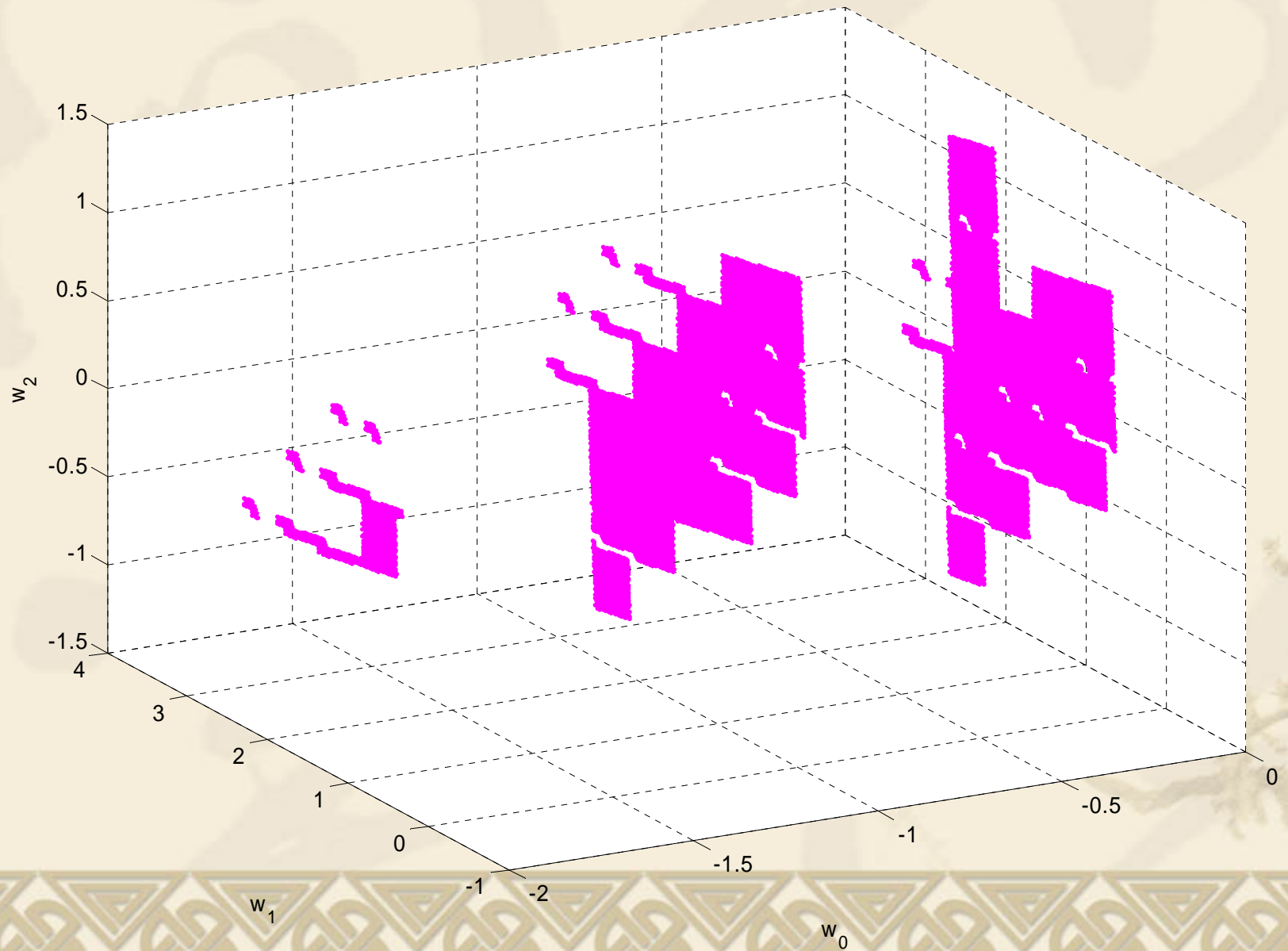


Nonlinear Systems

Assume that there are N training feature vectors and the dimension of the feature vectors is d . Denote $x_i(k)$ as the training features for $i=1,2,\dots,d$ and for $k=0,1,\dots,N-1$. Denote $\mathbf{x}(k)=[1, x_1(k), \dots, x_d(k)]^T$ as the training feature vectors for $k=0,1,\dots,N-1$. Denote $\mathbf{w}(k)=[w_0(k), w_1(k), \dots, w_d(k)]^T$ as the weights of the perceptrons, $t(k)$ as the desired output of the perceptron, $s(k)$ is the real output of the perceptron. Suppose that the weights of the perceptron is updated using the perceptron learning algorithm, then:

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \frac{t(k) - s(k)}{2} \mathbf{x}(k)$$

Nonlinear Systems



Challenge Problems and Some Solutions

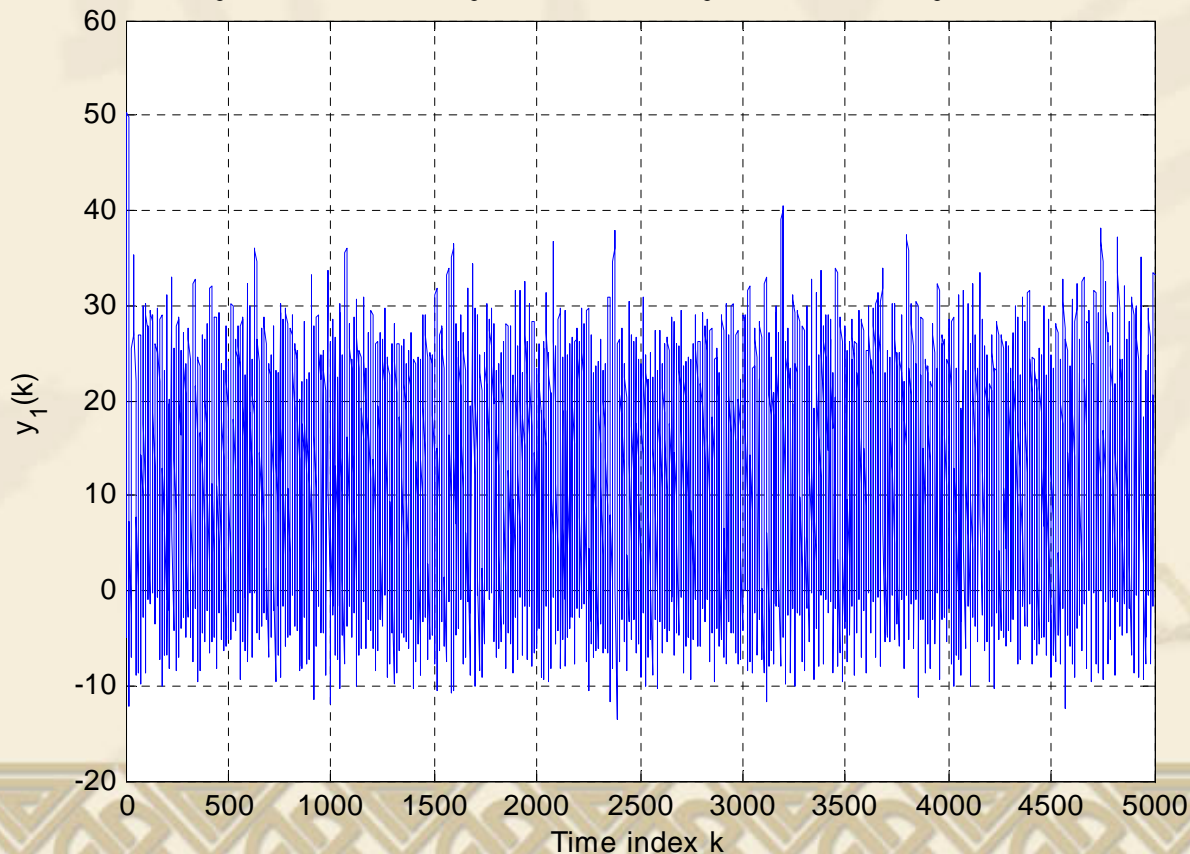
❖ Sigma delta modulators

∞ Stability depends on the initial condition.

$$F(z) = \frac{20z^{-1} - 74z^{-2} + 103.0497z^{-3} - 64.0015z^{-4} + 14.9584z^{-5}}{1 - 5z^{-1} + 10.0025z^{-2} - 10.0075z^{-3} + 5.0075z^{-4} - 1.0025z^{-5}}$$

$$\mathbf{x}(0) = [0, 0, 0, 0, 0]^T$$

$$u(k) = 0.75$$

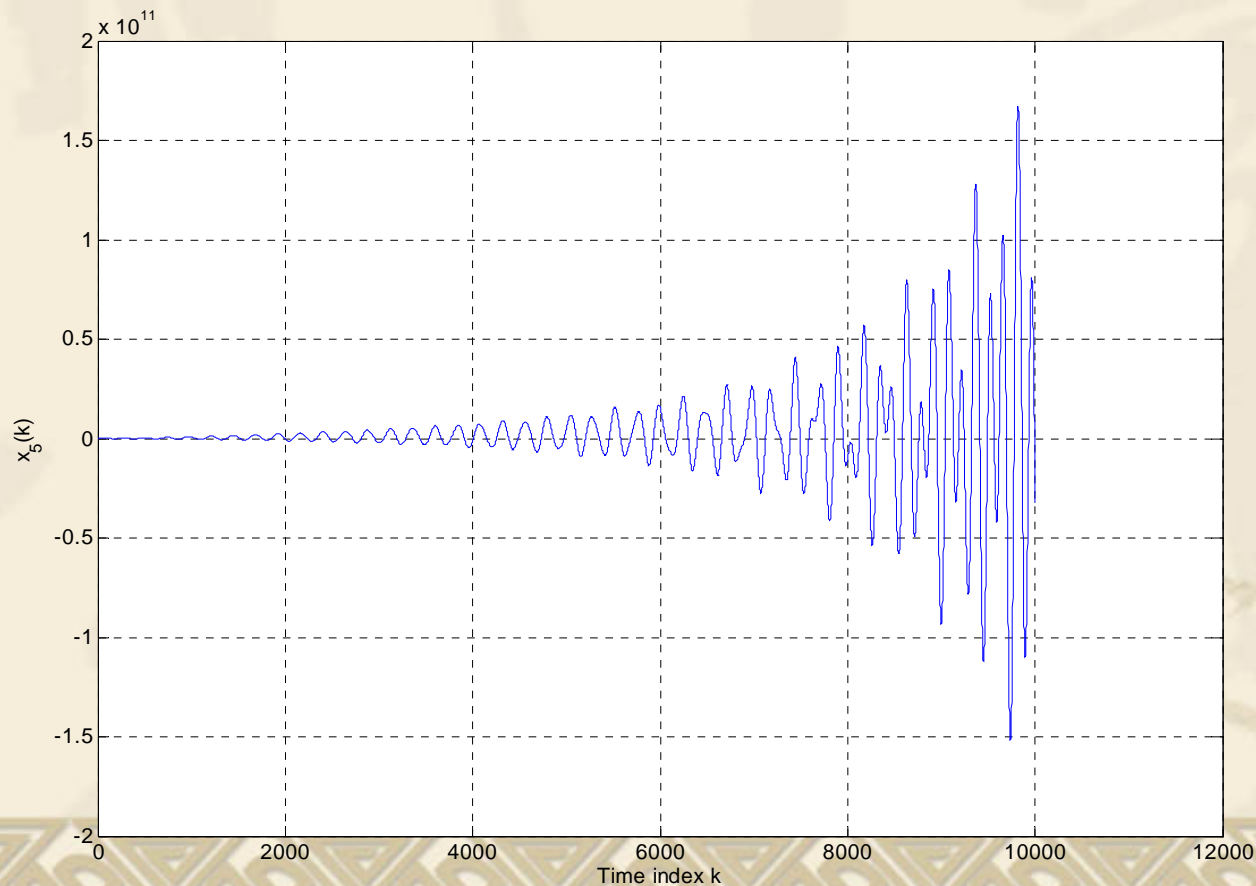


Challenge Problems and Some Solutions

❖ Sigma delta modulators

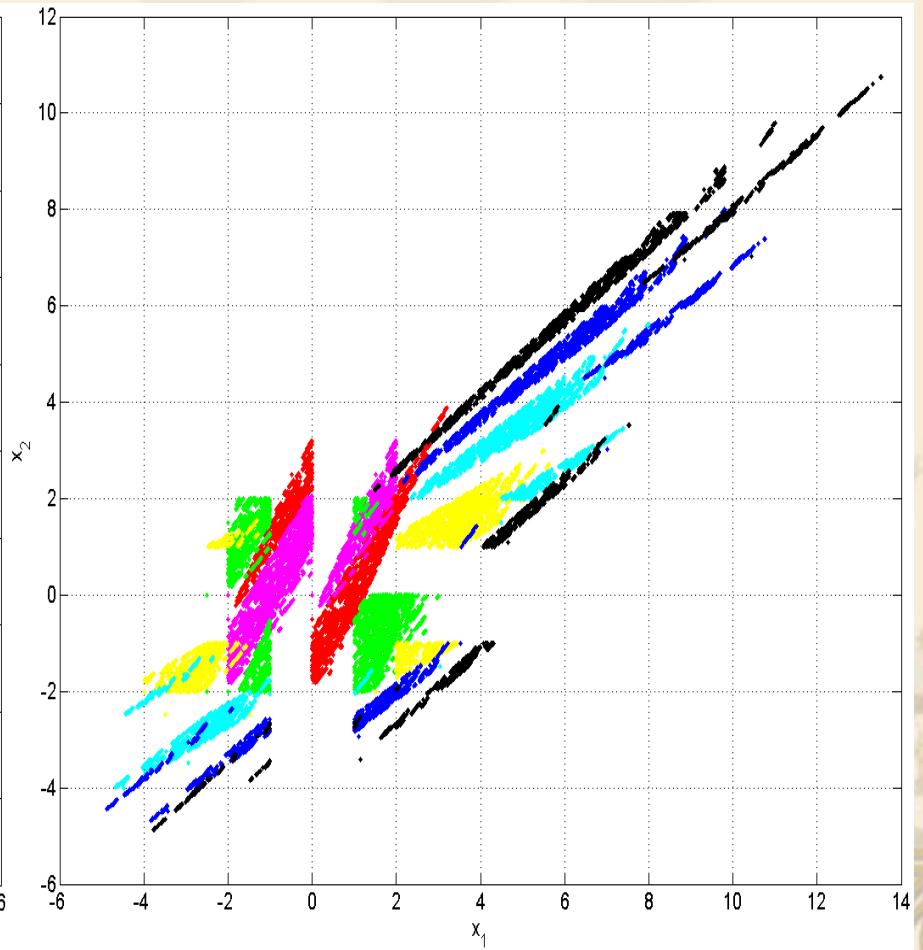
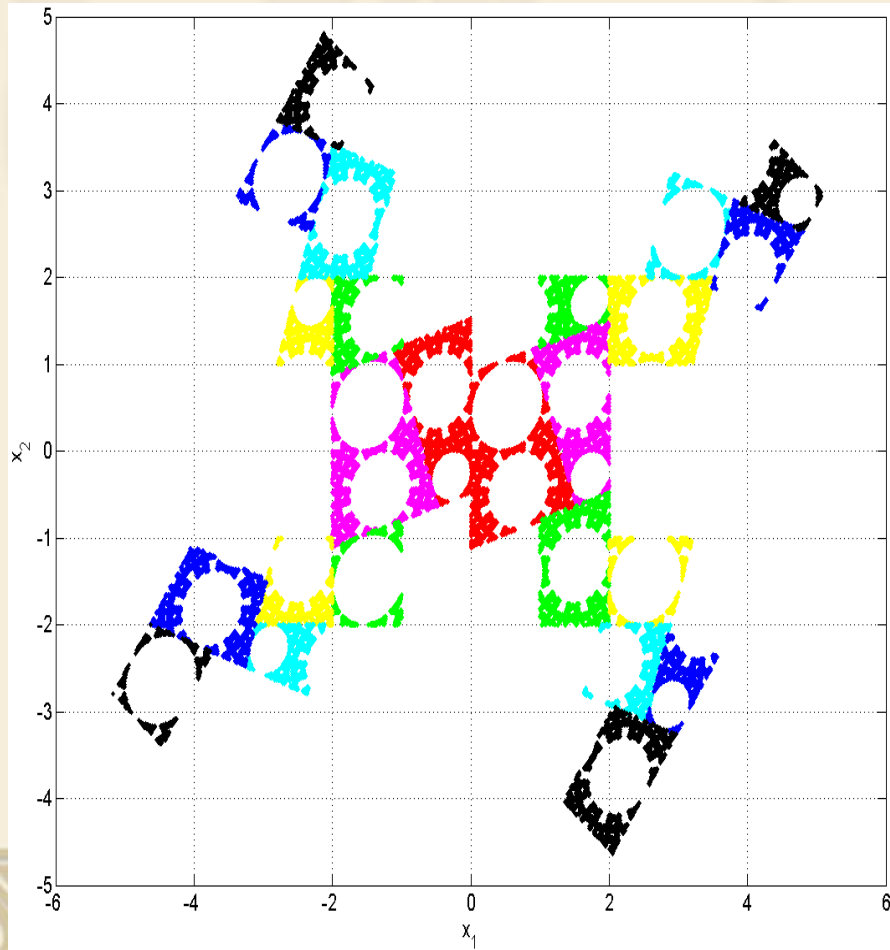
$$F(z) = \frac{20z^{-1} - 74z^{-2} + 103.0497z^{-3} - 64.0015z^{-4} + 14.9584z^{-5}}{1 - 5z^{-1} + 10.0025z^{-2} - 10.0075z^{-3} + 5.0075z^{-4} - 1.0025z^{-5}} \quad \mathbf{x}(0) = [0.001, 0, 0, 0, 0]^T$$

$$u(k) = 0.75$$



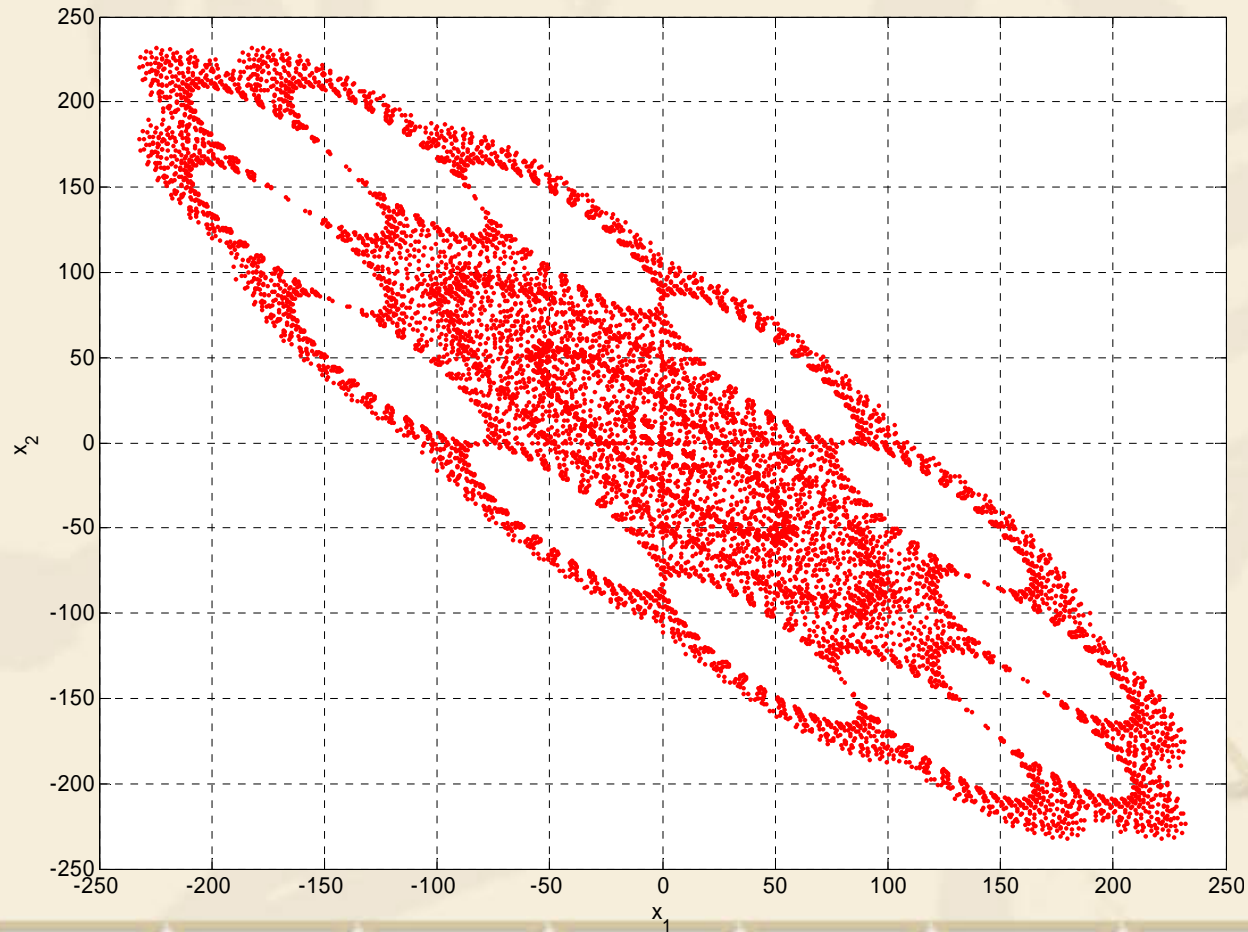
Challenge Problems and Some Solutions

❖ Sigma delta modulators



Challenge Problems and Some Solutions

❖ Sigma delta modulators



Challenge Problems and Some Solutions

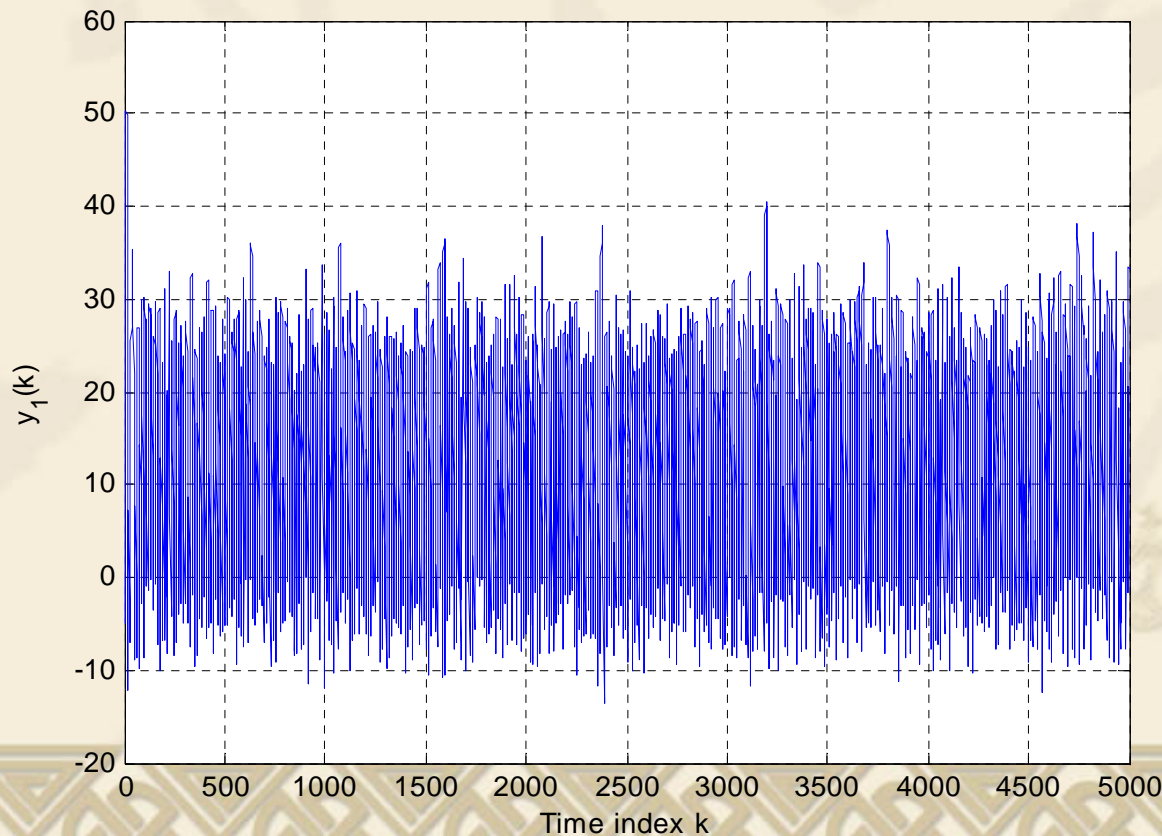
❖ Sigma delta modulators

∞ Stability depends on the input signals.

$$F(z) = \frac{20z^{-1} - 74z^{-2} + 103.0497z^{-3} - 64.0015z^{-4} + 14.9584z^{-5}}{1 - 5z^{-1} + 10.0025z^{-2} - 10.0075z^{-3} + 5.0075z^{-4} - 1.0025z^{-5}}$$

$$u(k) = 0.75$$

$$\mathbf{x}(0) = [0, 0, 0, 0, 0]^T$$



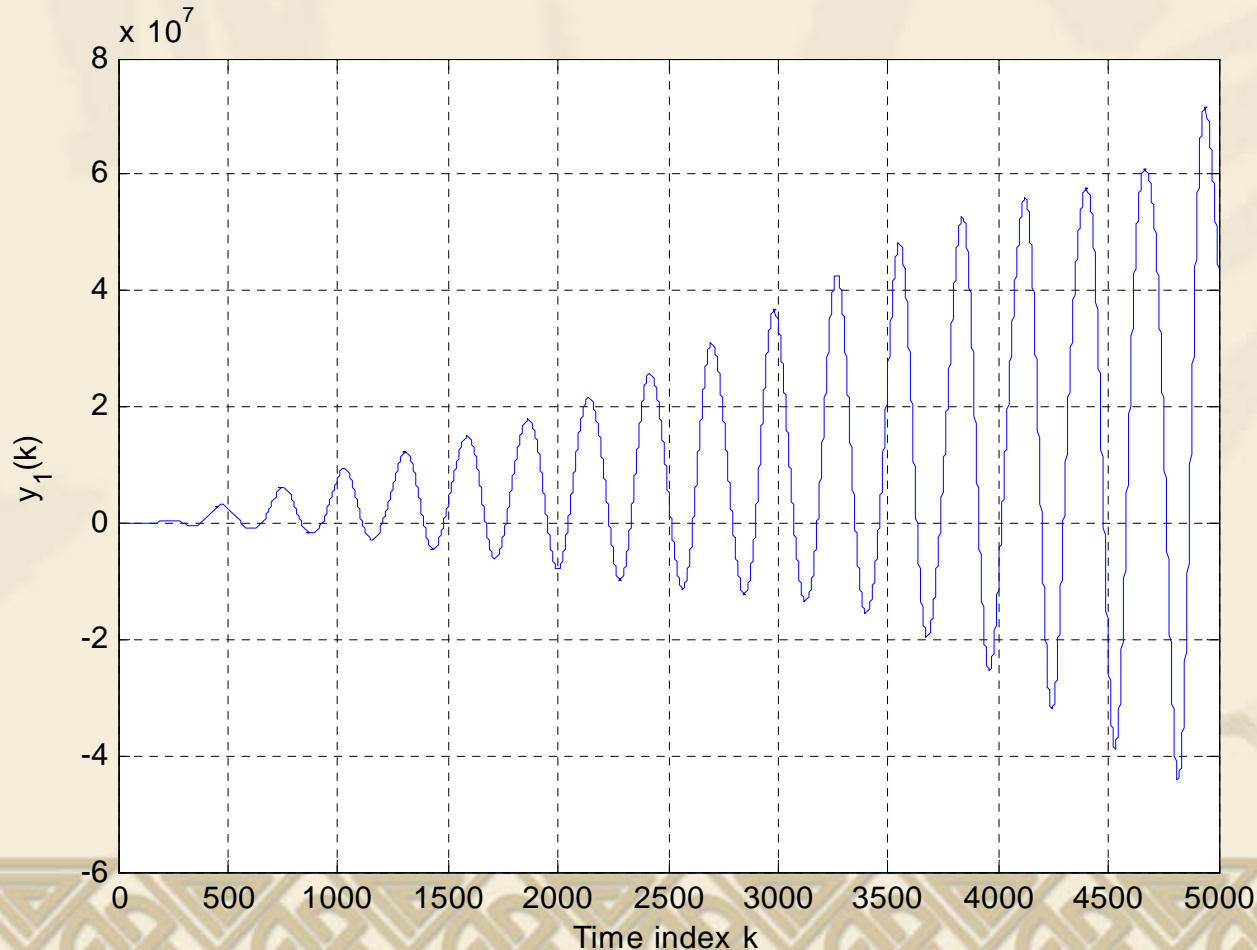
Challenge Problems and Some Solutions

❖ Sigma delta modulators

$$F(z) = \frac{20z^{-1} - 74z^{-2} + 103.0497z^{-3} - 64.0015z^{-4} + 14.9584z^{-5}}{1 - 5z^{-1} + 10.0025z^{-2} - 10.0075z^{-3} + 5.0075z^{-4} - 1.0025z^{-5}}$$

$$\mathbf{x}(0) = [0, 0, 0, 0, 0]^T$$

$$u(k) = 0.76$$



Challenge Problems and Some Solutions

❖ Sigma delta modulators

⌘ Suppose that $Q(a_N b_N) = -1$ and $|a_N| > |a_0|$. Define

$$\mathcal{P}_S \equiv \left\{ \mathbf{x} \equiv [x_1, \dots, x_N]^T \in \mathcal{P} : |x_1| < \left| \frac{2b_N}{a_N} \right| \right\}$$

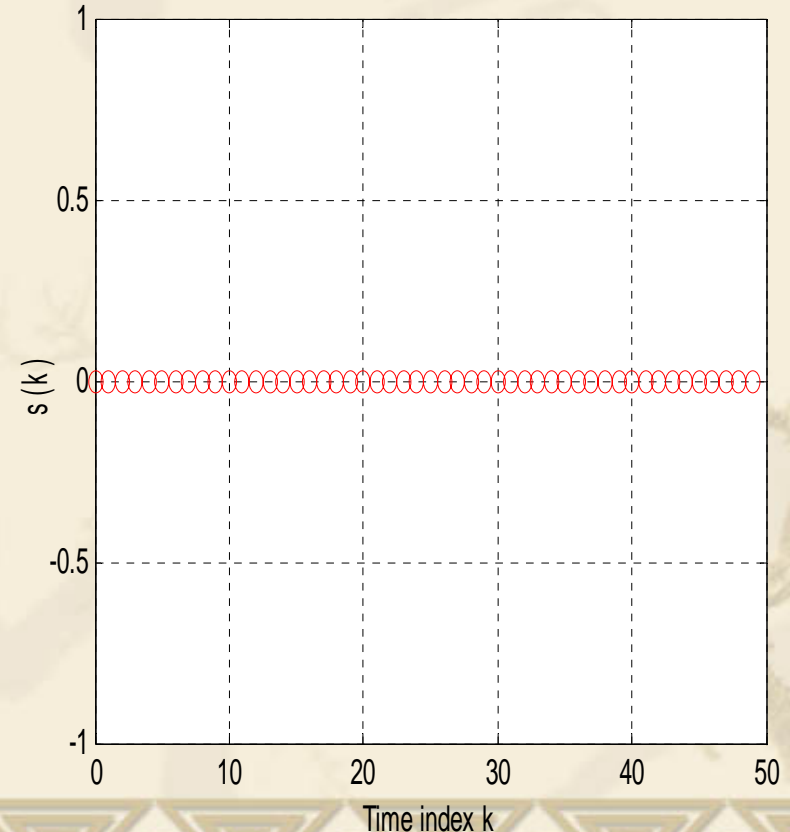
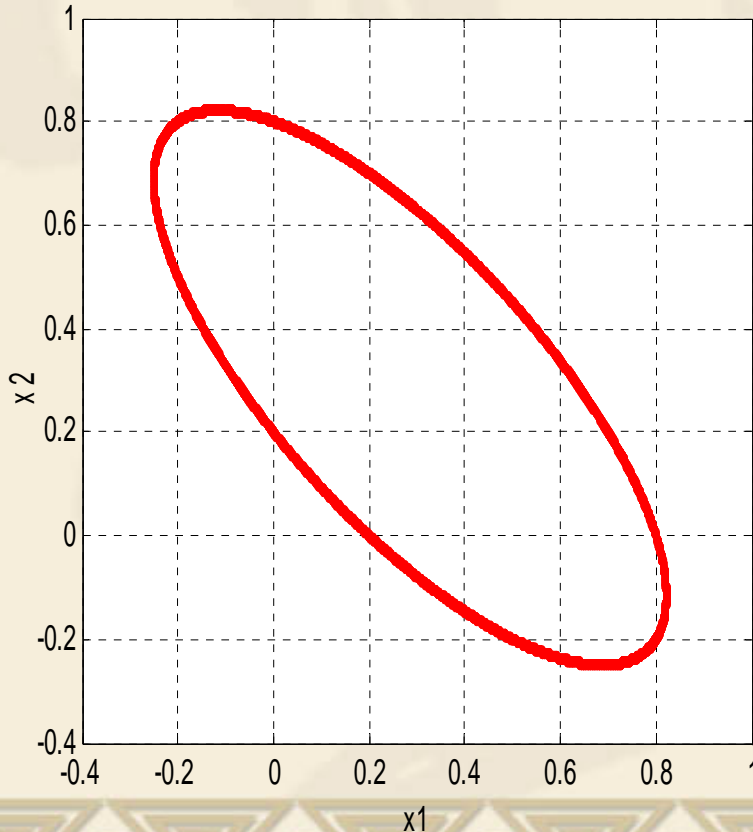
Assume that $\mathcal{P}_S \neq \emptyset$. Suppose that either $\frac{F(z)}{1 + KF(z)}$ is strictly stable,

or marginally stable and the frequency spectrum of the input of the loop filter does not contain an impulsive located at the natural frequency of the loop filter. If $\lim_{K \rightarrow 0^+} \frac{F(z)}{1 + KF(z)}$ is strictly stable for $K < \left| \frac{a_N}{2b_N} \right|$, then

the sigma delta modulator is globally stable.

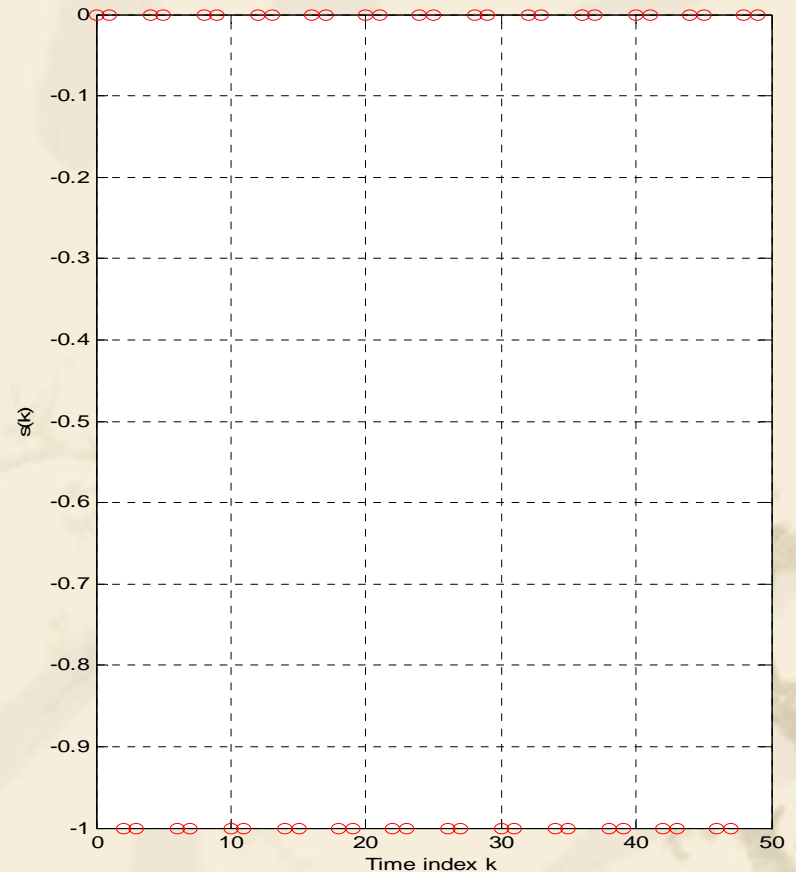
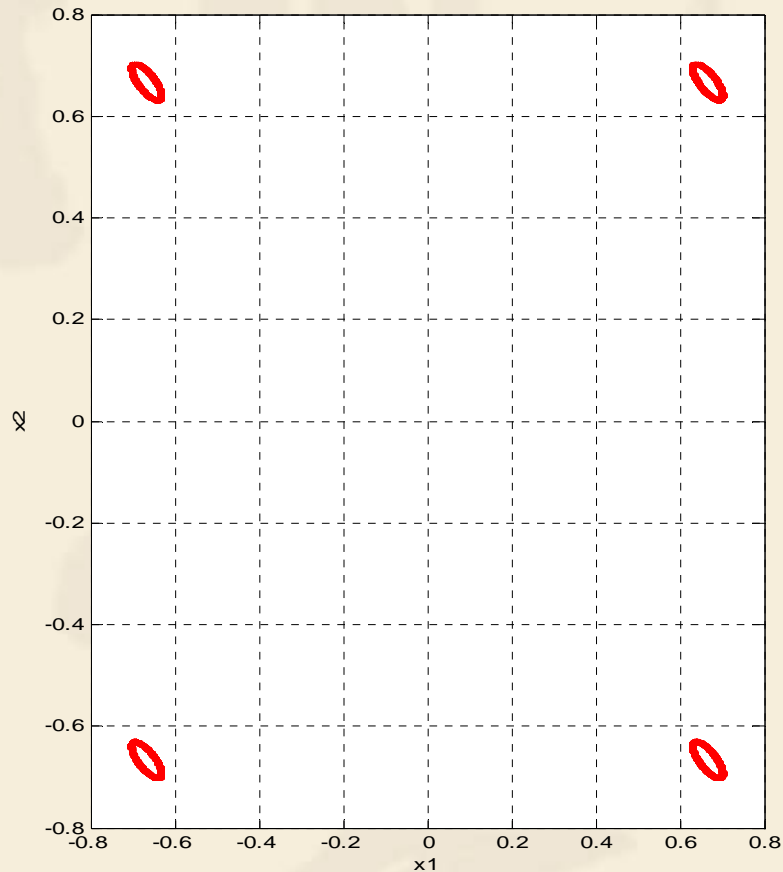
Challenge Problems and Some Solutions

- ❖ Digital filters with two's complement arithmetic
 - ∞ Digital filters with two's complement arithmetic could exhibit various behaviors.



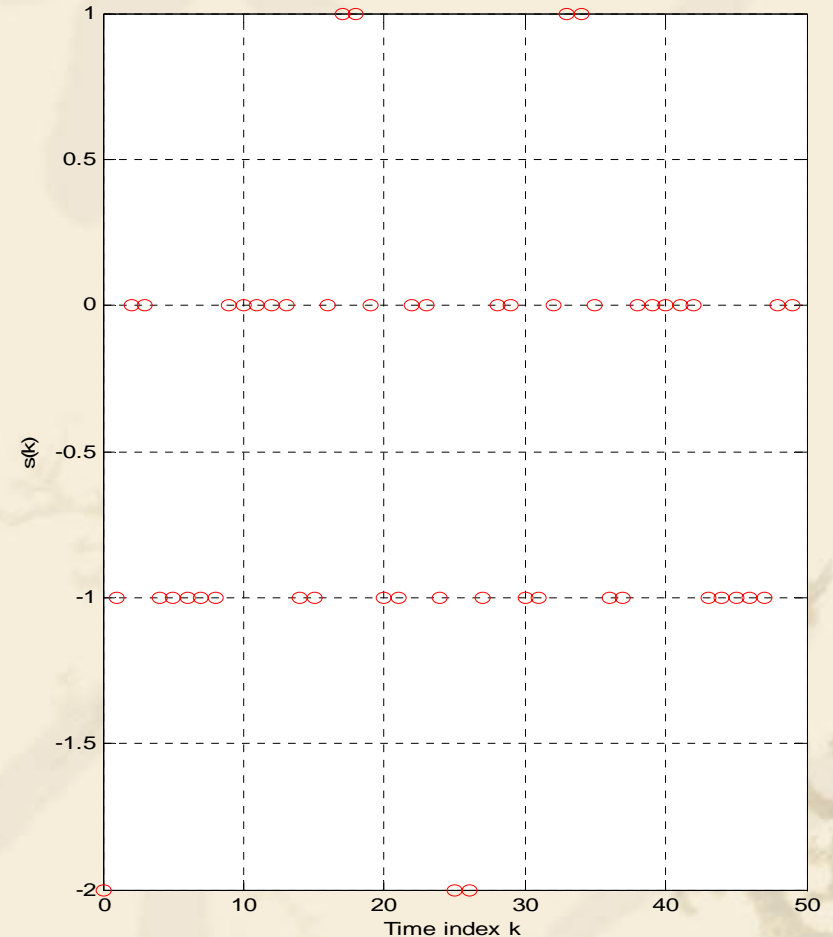
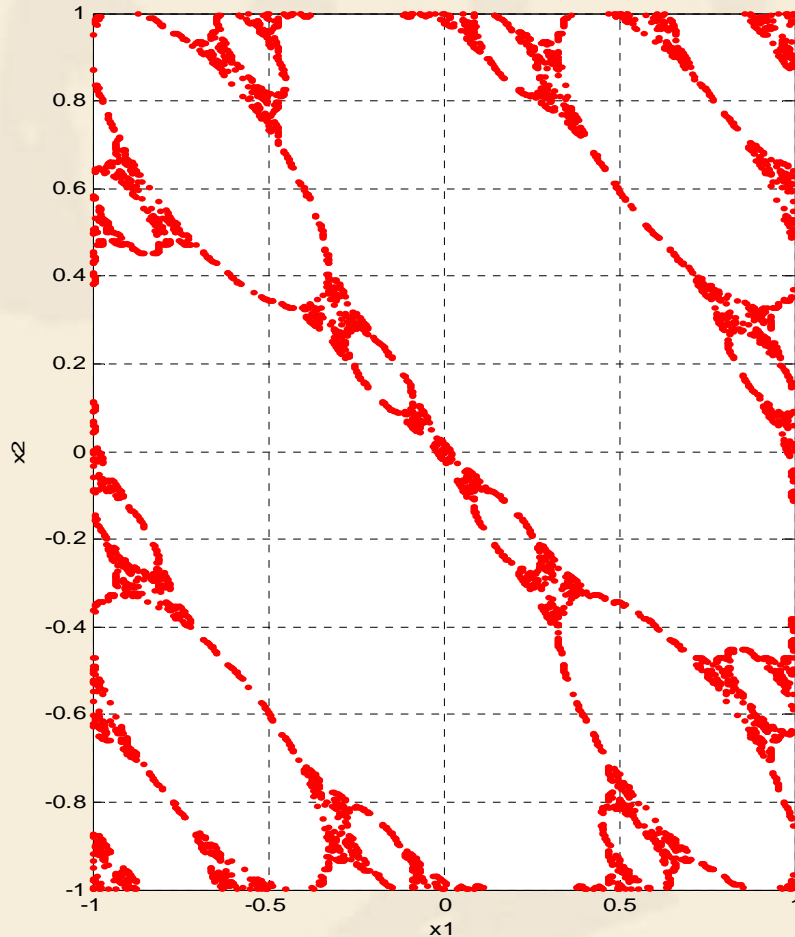
Challenge Problems and Some Solutions

- ❖ Digital filters with two's complement arithmetic



Challenge Problems and Some Solutions

❖ Digital filters with two's complement arithmetic



Challenge Problems and Some Solutions

❖ Digital filters with two's complement arithmetic

⌘ What are the conditions correspond to different behaviors?

⌘ Define $u(k) = c$ for $k \geq 0, c \in \mathbb{R}$
 $\cos \theta \equiv \frac{a}{2}$

$$\mathbf{T} \equiv \begin{bmatrix} 1 & 0 \\ \cos \theta & \sin \theta \end{bmatrix}$$

$$\hat{\mathbf{A}} \equiv \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$\mathbf{x}^* \equiv \frac{c}{2-a} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$s_0 \equiv s(0)$$

$$\hat{\mathbf{x}}(k) \equiv \mathbf{T}^{-1} \left(\mathbf{x}(k) - \frac{c + 2s_0}{2-a} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)$$

Challenge Problems and Some Solutions

- ❖ For the type I trajectory, the following three statements are equivalent each other:

$$\hat{\mathbf{x}}(k+1) = \hat{\mathbf{A}}\hat{\mathbf{x}}(k) \text{ for } k \geq 0$$

$$s(k) = s_0 \text{ for } k \geq 0$$

$$\mathbf{x}(0) \in \left\{ \mathbf{x}(0) : \left\| \mathbf{T}^{-1} \left(\mathbf{x}(0) - \frac{c + 2s_0}{c} \mathbf{x}^* \right) \right\| \leq 1 - \frac{|c + 2s_0|}{2 - a} \right\}$$

Challenge Problems and Some Solutions

- ❖ For the type II trajectory, the following three statements are equivalent each other:

$$\widehat{\mathbf{x}}_i(k+1) = \widehat{\mathbf{A}}^M \widehat{\mathbf{x}}_i(k) \text{ for } k \geq 0 \text{ and } i = 0, 1, \dots, M-1$$

$$\exists M \text{ such that } s(Mk+i) = s(i) \text{ for } k \geq 0 \text{ and } i = 0, 1, \dots, M-1$$

$$\mathbf{x}(0) \in \left\{ \mathbf{x}(0) : \left\| \mathbf{T}^{-1}(\mathbf{x}(i) - \mathbf{x}_i^*) \right\|_{\infty} \leq 1 - \left\| \mathbf{x}_i^* \right\|_{\infty} \right\} \text{ for } i = 0, 1, \dots, M-1$$

Challenge Problems and Some Solutions

∞ For the type III trajectory, the following three statements are equivalent each other:

❖ There is an elliptical fractal pattern exhibited on the phase plane.

❖ The symbolic sequences are aperiodic.

❖ The set of initial conditions is $I^2 \setminus \bigcap_{\forall M} D_M$
where

$$D_M = \left\{ \mathbf{x}(0) : \left\| \mathbf{T}^{-1}(\mathbf{x}(i) - \mathbf{x}_i^*) \right\| \leq 1 - \left\| \mathbf{x}_i^* \right\|_{\infty} \text{ and } s(i) = s(i + Mk) \right\}$$

which is also an elliptical fractal set.

Challenge Problems and Some Solutions

❖ Perceptrons

- ❧ Suppose that the set of training feature vectors are linearly separable. Then, by using the perceptron training algorithm, the weights of the perceptrons will converge to a fixed point.
- ❧ When the set of training feature vectors are not linearly separable, will the weights be bounded?
- ❧ If it is bounded, under what conditions will the weights exhibit limit cycle behaviors and under what conditions will the weights exhibit chaotic behaviors?

Challenge Problems and Some Solutions

❖ Perceptrons

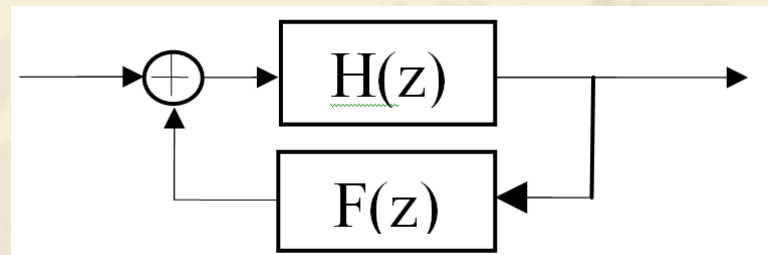
❧ If $\exists \mathbf{w}^*(0) \in \mathcal{R}^{d+1}$ and $\exists \tilde{B} \geq 0$ such that $\|\mathbf{w}^*(k)\| \leq \tilde{B} \quad \forall k \geq 0$,
then $\exists B'' \geq 0$ such that $\|\mathbf{w}(k)\| \leq B'' \quad \forall k \geq 0$ and $\forall \mathbf{w}(0) \in \mathcal{R}^{d+1}$

❧ Suppose that q_1 and q_2 are co-prime and M are N integers. That is $q_1 M = q_2 N$. Then $\mathbf{w}^*(n)$ is periodic with period M if and only if

$$\sum_{j=0}^{M-1} \frac{t(kM + j) - Q\left(\left(\mathbf{w}^*(kM + j)\right)^T \mathbf{x}(kM + j)\right)}{2} \mathbf{x}(kM + j) = \mathbf{0}$$

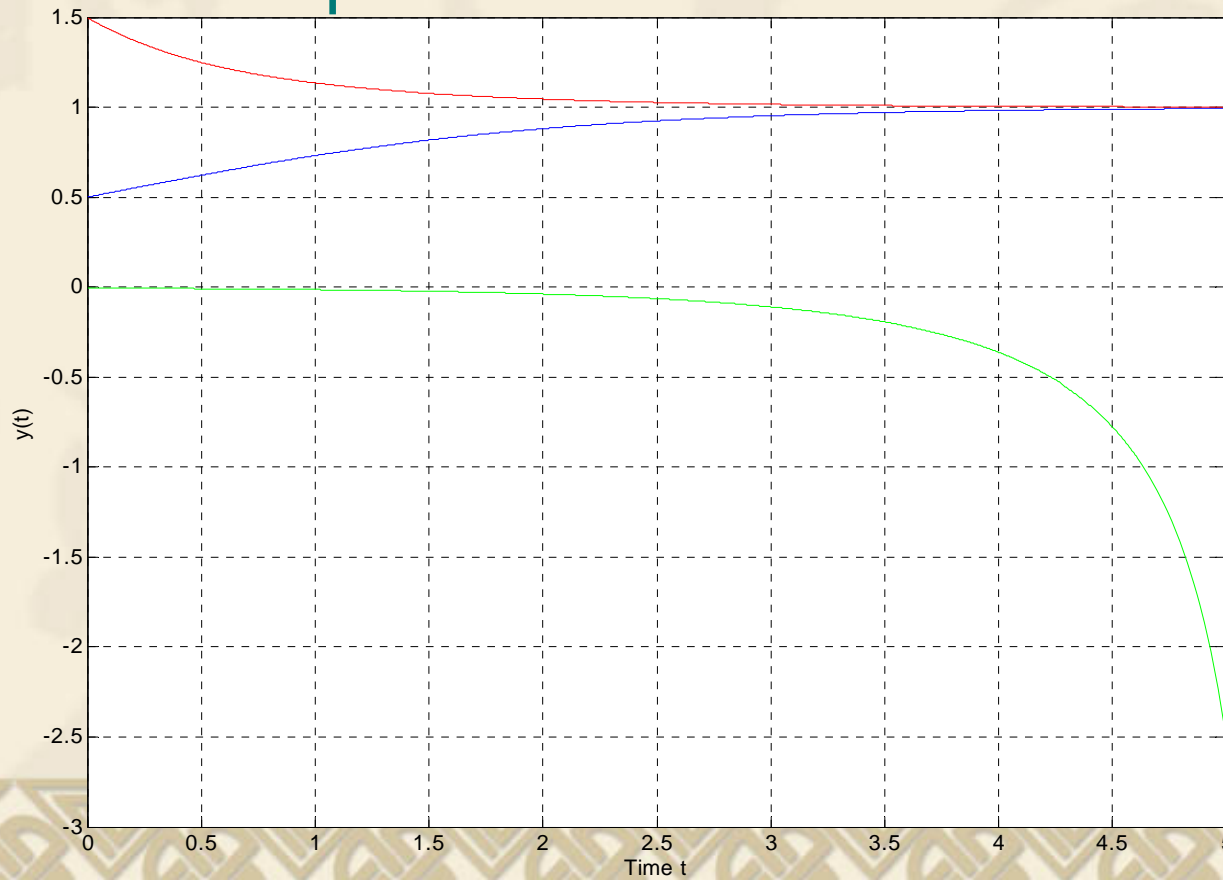
Impulsive Control

- ❖ Conventional controller generates control signals fed into the input of the plant.
- ❖ If the plant is not controllable, then the plant will lose control.
- ❖ The control force usually lasts forever.



Impulsive Control

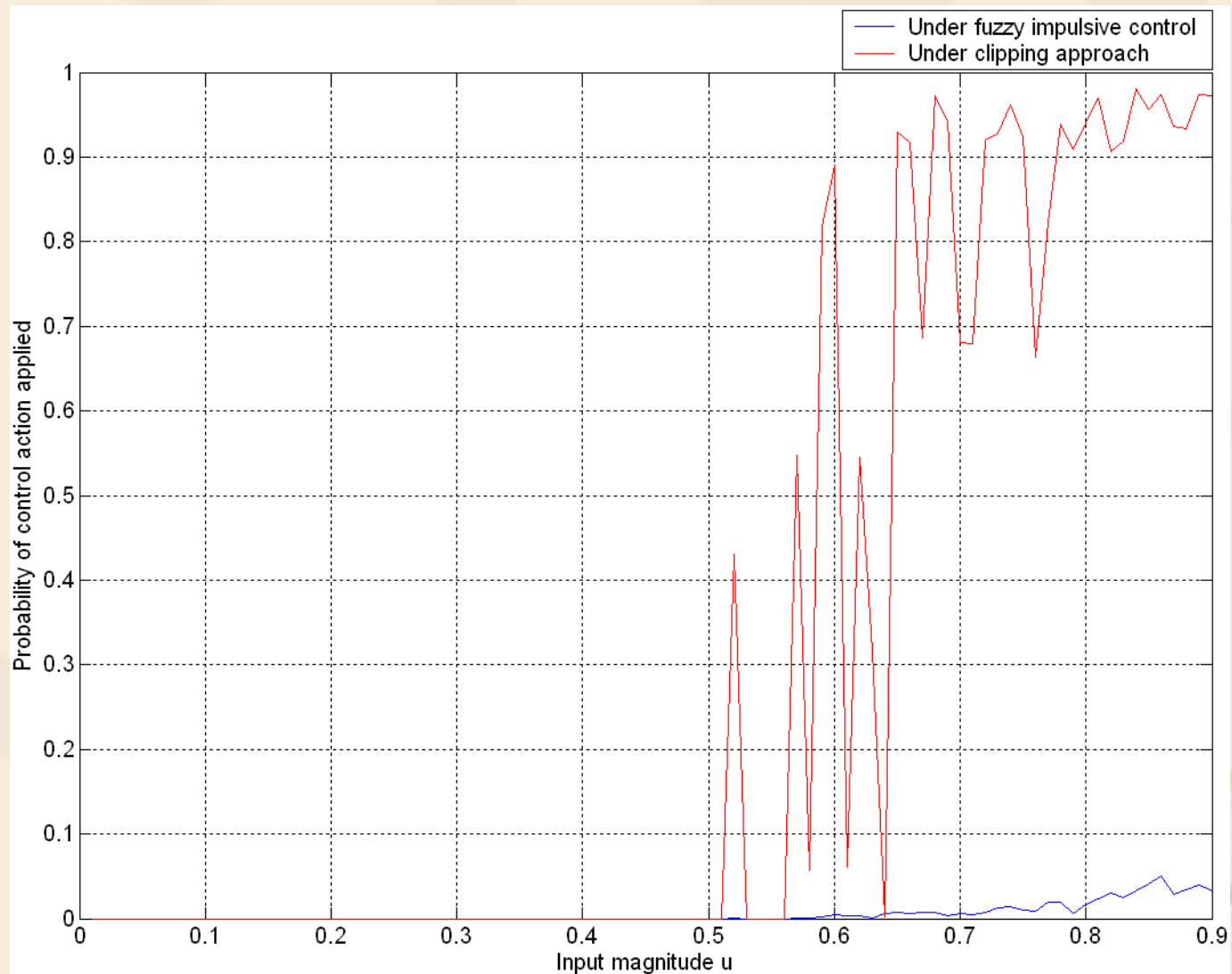
- ❖ Since different initial conditions may correspond to different stability conditions, if the initial conditions are moved to another positions, then the plant will be automatically stablized and the control force can be removed from the plant.



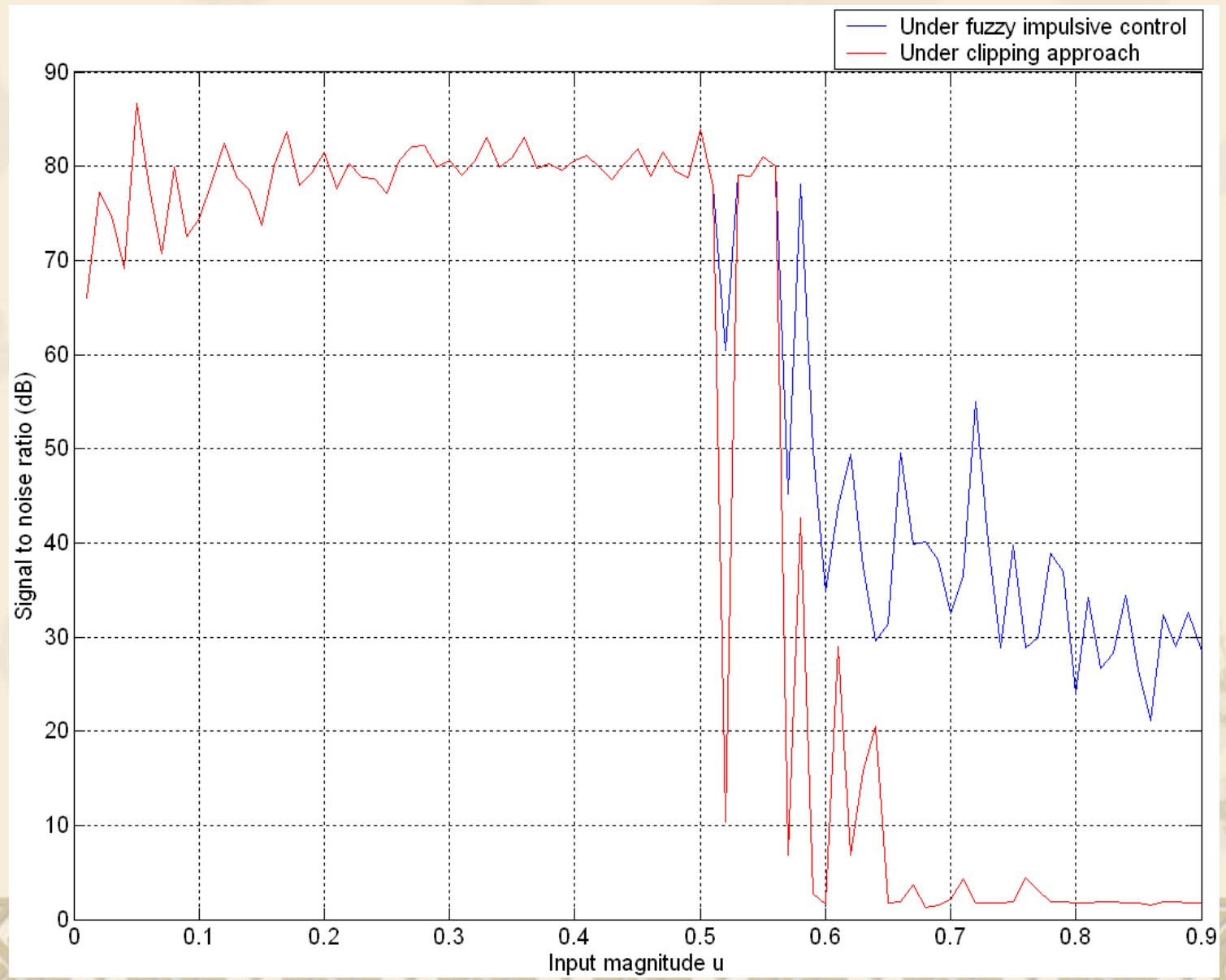
Impulsive Control

- ❖ Control signals are not fed into the input of the plant, so controllability is not an issue.
- ❖ The impulsive controller is usually implemented via a reset circuit.

Impulsive Control



Impulsive Control



Fuzzy Control

- ❖ In crisp set, either $x \in A$ or $x \notin A$.
- ❖ A fuzzy set is a set that $x \in A$ associated with a fuzzy membership function $\mu_A(x) \in [0,1]$. If $\mu_A(x)=0$, then $x \notin A$. If $\mu_A(x)=1$, then $x \in A$.
- ❖ In the traditional binary logics, all combinational logics can be represented by combinations of complement, intersection and union of binary variables because all Boolean equations can be represented by sum of products or product of sums.
- ❖ Similarly, there are three fundamentals operations in fuzzy logics and these operations represent the traditional complement, intersection and union operations.

Fuzzy Control

- ❖ For the fuzzy complement operations, the fuzzy set A maps to a fuzzy set \bar{A} with the fuzzy membership function $\mu_{\bar{A}}(x) \equiv c(\mu_A(x))$ satisfying the properties $c(0)=1$, $c(1)=0$, and $c(x_1) > c(x_2)$ if $x_1 < x_2$. The common fuzzy complement operation are Sugeno operations

$$c_{\lambda}(a) \equiv \frac{1-a}{1+\lambda a} \text{ for } \lambda \geq -1.$$

- ❖ For the fuzzy intersection operations, the fuzzy sets A and B maps to a fuzzy set $A \cap B$ with the fuzzy membership function $\mu_{A \cap B}(x) \equiv t(\mu_A(x), \mu_B(x))$ satisfying $t(0,0)=0$, $t(1,a)=t(a,1)=a$, $t(a,b)=t(b,a)$, $t(t(a,b),c)=t(a,t(b,c))$ and $t(a,b) \leq t(a',b')$ if $a \leq a'$ and $b \leq b'$. The common fuzzy intersection operations are $t(a,b)=ab$.

Fuzzy Control

- ❖ For the fuzzy union operations, the fuzzy sets A and B maps to a fuzzy set $A \cup B$ with the fuzzy membership function $\mu_{A \cup B}(x) \equiv s(\mu_A(x), \mu_B(x))$ satisfying $s(1,1)=1$, $s(0,a)=s(a,0)=a$, $s(a,b)=s(b,a)$, $s(s(a,b),c)=s(a,s(b,c))$ and $s(a,b) \leq s(a',b')$ if $a \leq a'$ or $b \leq b'$. The common fuzzy union operation is $s(a,b)=\max(a,b)$.
- ❖ A fuzzy relationship Q in $U_1 \times U_2 \times \dots \times U_n$ is defined as $Q \equiv \{(u_1 \times u_2 \times \dots \times u_n), \mu_Q(u_1 \times u_2 \times \dots \times u_n) : (u_1 \times u_2 \times \dots \times u_n) \in U_1 \times U_2 \times \dots \times U_n\}$, where $\mu_Q: U_1 \times U_2 \times \dots \times U_n \rightarrow [0,1]$.

Fuzzy Control

- ❖ If a variable can take words in natural languages as its values, it is called a linguistic variable, where the words are characterized by fuzzy sets. A linguistic variable is characterized by (X, T, U, M) , where X is the name of the linguistic variable, T is the set of linguistic values that X can take, U is the actual physical domain in which the linguistic variable X takes its quantitative values, and M is a set of semantic rules which relates each linguistic values in T with a fuzzy set.

Fuzzy Control

- ❖ For an example, X is the speed of a car, $T=\{\text{slow, medium, fast}\}$, $U=[0, V_{\max}]$, and M consists of three fuzzy membership functions that describes slow, medium and fast.
- ❖ A fuzzy if then rule is a conditional statement expressed in the form of “IF fuzzy proposition 1 (FP_1), then fuzzy proposition 2 (FP_2).”.
- ❖ There are two types of fuzzy propositions, an atomic fuzzy proposition and a compound fuzzy proposition.
- ❖ An atomic fuzzy proposition is a single statement “ x is A ”, in which x is a linguistic variable and A is a linguistic value of x .
- ❖ A compound fuzzy proposition is a composition of atomic fuzzy propositions using the connectives “and”, “or” and “not” which represent fuzzy intersection, fuzzy union and fuzzy complement, respectively. For an example, “ x is A and x is not B .” The fuzzy if then rule is a fuzzy implication. The common fuzzy implications are Mamdani implication

$$\mu_{Q_{MM}}(x, y) \equiv \min(\mu_{FP_1}(x), \mu_{FP_2}(y))$$

Fuzzy Control

- ❖ A fuzzy system usually consists of three parts: fuzzifier, fuzzy engine and defuzzifier.
- ❖ A fuzzifier is a system that maps the inputs of a system to fuzzy inputs associated with fuzzy member functions.
- ❖ A fuzzy engine is a system that maps the input fuzzy membership functions to output member functions.
- ❖ A defuzzifier is a system that maps the fuzzy outputs associated with fuzzy member functions to outputs of the system.

Fuzzy Control

- ❖ There are three common types of fuzzy systems, pure fuzzy systems, Takagi-Sugeno-Kang (TSK) fuzzy systems and fuzzy systems with fuzzifier and defuzzifier.
- ❖ The pure fuzzy systems map the input fuzzy sets to output fuzzy sets via a fuzzy inference engine which consists of a set of fuzzy if then rules.
- ❖ The TSK fuzzy systems formulate the fuzzy if then rules via a weighted average fuzzy inference engine.
- ❖ Fuzzy systems with fuzzifier and defuzzifier employ the fuzzifier to map the real valued variables into fuzzy sets and the defuzzifier transforms fuzzy sets into real valued variables.

Conclusions

- ❖ Dynamics of nonlinear systems depend on initial conditions and input signals.
- ❖ Chaotic systems is sensitive to initial conditions, topological transitive and consisting of rich frequency spectra.
- ❖ Symbolic dynamics is a kind of dynamics that signals in the systems are multi-levelled.
- ❖ Many real systems, such as digital filters with two's complement arithmetic, sigma delta modulators and perceptrons, are governed by symbolic dynamics.
- ❖ Global stability conditions of sigma delta modulators and boundedness conditions of perceptrons are discussed. Conditions for digital filters with two's complement arithmetic exhibiting various behaviors are presented.
- ❖ Impulsive control is to generate an impulsive or reset the system states directly so that no further control action is required and the systems will be automatically stabilized.
- ❖ Fuzzy control to control the systems via fuzzy rules.

Questions and Answers



Thank you!

Let me think...